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ADVERTISEMENT.

THE Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the council-books and journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries, till the Forty-seventh Volume: the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March, 1752. And the grounds of their choice are, and will continue to

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be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgment of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report, and even to certify in the public news-papers, that they have met with the highest applause and approbation. And therefore it is hoped, that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

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PHILOSOPHICAL

TRANSACTIONS.

I. *On the effects of temperature on the intensity of magnetic forces; and on the diurnal variation of the terrestrial magnetic intensity.* *By* SAMUEL HUNTER CHRISTIE, *Esq. M. A. of Trinity College, Cambridge, Fellow of the Cambridge Philosophical Society: of the Royal Military Academy. Communicated by the President.*

Read June 17, 1824.

IN the paper on the diurnal deviations of the horizontal needle when under the influence of magnets, which the President did me the honour to present, I stated that these deviations were partly the effects of changes that took place in the temperature of the magnets; and that although the conclusions which I drew from the observations respecting the increase and decrease of the terrestrial magnetic forces during the day would not be materially affected, it was my intention to undertake a series of experiments for the purpose of determining the precise effects of changes of temperature in the magnets, so as to be able to free the observations entirely from such effects.

These experiments were immediately made; but I was induced from some effects which I observed, to carry them to

a greater extent, in the scale of temperature, than was necessary for the object which I had at first in view. In consequence of this, and the length of the calculations into which I have been obliged to enter, the accomplishment of my purpose was delayed for a considerable time, and continued indisposition has since prevented me, until now, completing the arrangement of the tables of results.

In the present paper, I propose to detail the experiments which I made in order to determine the effect of changes of temperature on the forces of the magnets, to the extent to which I observed their temperature to vary, during my observations on the diurnal changes in the direction of the needle, when under their influence; to apply the results which I obtained to the correction of the observations themselves, thereby accounting for the apparent anomalies noticed by Mr. BARLOW and myself, in the observations made in doors and in the open air; and by means of these corrected observations, to point out the diurnal variations in the terrestrial magnetic intensity.

It had been my intention to determine purely from observation the portion of the arc of deviation due to the changes which I noticed in the temperature of the magnets; but I found that this depended so much on the situation of the point at which the needle was held in equilibrio by the terrestrial forces and those of the magnets, that it would hardly be possible to determine how much of this portion was due to the extent of the change of temperature, or the degree of temperature where the change took place, and how much to the azimuth of the needle, when affected by this change. I was therefore under the necessity of having recourse to

theory, and adopted the simplest, and that which is most generally received, viz. that the forces which two magnets exert upon one another may be referred to two centres or poles in each, near their respective ends; and that for either pole in one of the magnets, one pole of the other magnet is urged towards it, and the other from it, by forces varying inversely as the squares of their respective distances from that pole. Of the correctness of this theory of the action of one magnet upon another, the conclusions which I have obtained have given me no reason to doubt.

In the observations on the diurnal changes in the positions of the points of equilibrium at which the pole of the needle was retained by the joint action of two magnets and the terrestrial magnetism, where I noted the changes that took place in the temperature of the magnets, to which observations I have alluded near the conclusion of my former paper, two magnets, as in several of the preceding observations, were placed, with their axes in the magnetic meridian, on the same horizontal table as the compass, at equal distances from the centre of the needle, one towards the north, the other towards the south, the north pole of each magnet being towards the north; and their distances from the centre were such, that the points of equilibrium were nearly 180°, or south, N. 80° E. and N. 80° W. To determine here the changes that would take place in the situation of these points from changes in the force of the magnets, arising from a variation of their temperature, it was first necessary to determine the changes in the forces themselves, arising from certain variations of the temperature of the magnets, by observing the corresponding changes in the direction of the needle.

To obtain the equation requisite for this purpose, take the centre of the needle as the origin of the rectangular co-ordinates, the axis of the x's being in the magnetic meridian. Let x and y be the co-ordinates to the south pole of the needle, x being measured towards the north, and y towards the west: also, let r be the distance of either pole of the needle from its centre; ρ the distance of the poles of each of the magnets from their respective centres; and R the distance between the centre of the needle and the centre of either magnet. For the sake of expressing clearly and concisely the distances between the poles of the needle and those of the magnets, we will indicate these points as follow:

s, the *south* pole of the needle; that is, the pole which, when the needle is freely suspended, points towards the *north*;

n, the *north* pole of the needle;

σ_N, the *south* pole of the magnet which is to the *north* of the needle; that is, its pole *nearest* to the centre of the needle;

ν_N, the *north* pole of the same magnet, or its pole which is *furthest* from the needle's centre;

σ_S, the *south* pole of the *south* magnet, or that pole which is *furthest* from the centre of the needle;

ν_S, the *north* pole of the same magnet, or that pole which is *nearest* to the centre of the needle.

Now resolve the terrestrial magnetic force acting on the north arm of the needle, in the line of the dip, into two; one horizontal or in the direction x, and the other vertical: and let the horizontal force be M. Also, let the force with which a pole of the needle is repelled from the pole of the same name of either magnet, or attracted towards that of a con-

trary name at the unity of distance, be F: then the forces acting on the south pole of the needle will be,

M in the direction x;

$-\frac{F}{(s\sigma_N)^2}$, in the direction $s\sigma_N$; $\frac{F}{(s\nu_N)^2}$, in the direction $s\nu_N$;

$-\frac{F}{(s\sigma_S)^2}$, in the direction $s\sigma_S$; $\frac{F}{(s\nu_S)^2}$, in the direction $s\nu_S$.

The north pole of the needle will be urged by forces equal and parallel to these, but in contrary directions; so that in investigating the conditions of equilibrium of the needle, we may consider only the equilibrium of the south pole urged by forces double of the preceding, and constrained to move in a circle; and it is evident that the equation of equilibrium will be the same, whether we take these forces, or the doubles of them.

Resolving these forces into others in the directions x and y, calling X the sum of all the forces in the direction x, and Y the sum of all the forces in the direction y, we shall have,

$$X = M - F\left\{\frac{R-\rho-x}{(s\sigma_N)^3} - \frac{R+\rho-x}{(s\nu_N)^3} - \frac{R+\rho+x}{(s\sigma_S)^3} + \frac{R-\rho+x}{(s\nu_S)^3}\right\}$$

$$Y = F\cdot\left\{\frac{y}{(s\sigma_N)^3} - \frac{y}{(s\nu_N)^3} + \frac{y}{(s\sigma_S)^3} - \frac{y}{(s\nu_S)^3}\right\}$$

The general equation of equilibrium for a point acted upon by forces in the same plane, and constrained to move in a curve whose equation is $L = o$, is

$$X\,dx + Y\,dy + \lambda\,dL = o. \qquad (1)$$

From this we obtain

$$X + \lambda\cdot\frac{dL}{dx} = o, \text{ and } Y + \lambda\frac{dL}{dy} = o;$$

whence

$$X\cdot\frac{dL}{dy} - Y\cdot\frac{dL}{dx} = o \qquad (2)$$

The equation $L = o$ is in this case,

$$x^2 + y^2 - r^2 = o;$$

and consequently $\frac{dL}{dx} = 2x$, $\frac{dL}{dy} = 2y$.

The equation (2) therefore becomes

$$Xy - Yx = o.$$

Substituting in this equation the values previously found for X and Y, and dividing by y, we obtain

$$M - F.\left[(R-\rho)\cdot\left\{\frac{1}{(s\sigma_N)^3}+\frac{1}{(s\nu_S)^3}\right\}-(R+\rho)\cdot\left\{\frac{1}{(s\nu_N)^3}+\frac{1}{(s\sigma_S)^3}\right\}\right]=o \ldots\ldots\ldots (A)$$

Let φ be the angle which the axis of the needle makes with the meridian, or the azimuth of the point of equilibrium, and we shall have,

$$(s\sigma_N)^2=(R-\rho)^2+r^2-2r(R-\rho)\cos.\varphi;\ (s\nu_S)^2=(R-\rho)^2+r^2+2r(R-\rho)\cdot\cos.\varphi;$$

$$(s\nu_N)^2=(R+\rho)^2+r^2-2r(R+\rho)\cos.\varphi;\ (s\sigma_S)^2=(R+\rho)^2+r^2+2r(R+\rho)\ \cos.\varphi.$$

Substituting these values in the equation (A), it becomes,

$$M - F.\left\{\begin{array}{l}\dfrac{R-\rho}{\left\{(R-\rho)^2+r^2\right\}^{\frac{3}{2}}}\cdot\left\{\dfrac{1}{\left\{1-\frac{2r(R-\rho)}{(R-\rho)^2+r^2}\cdot\cos.\varphi\right\}^{\frac{3}{2}}}+\dfrac{1}{\left\{1+\frac{2r(R-\rho)}{(R-\rho)^2+r^2}\cdot\cos.\varphi\right\}^{\frac{3}{2}}}\right\}\\ -\dfrac{R+\rho}{\left\{(R+\rho)^2+r^2\right\}^{\frac{3}{2}}}\cdot\left\{\dfrac{1}{\left\{1-\frac{2r(R+\rho)}{(R+\rho)^2+r^2}\cdot\cos.\varphi\right\}^{\frac{3}{2}}}+\dfrac{1}{\left\{1+\frac{2r(R+\rho)}{(R+\rho)^2+r^2}\cdot\cos.\varphi\right\}^{\frac{3}{2}}}\right\}\end{array}\right\}=o \ldots\ldots (B)$$

From this equation the value of F in terms of M may be found for any values of φ, the distances R, r and ρ being known; and if we suppose M constant during the observations, the variations in the intensity of the force F may be obtained from the observed variations in the value of φ.

If the angle φ does not differ from a right angle by more than 10° or even 20°, by expanding the several fractions, no sensible error will arise by limiting the series to a few of the

first terms, and we shall in these cases thus obtain a much more convenient equation for computation. Since

$$\frac{1}{(1-a\cos.\varphi)^{\frac{3}{2}}} = 1 + \frac{3}{2}.a\cos.\varphi + \frac{3\cdot5}{2\cdot4}a^2\cos.^2\varphi + \frac{3\cdot5\cdot7}{2\cdot4\cdot6}a^3\cos.^3\varphi + \frac{3\cdot5\cdot7\cdot9}{2\cdot4\cdot6\cdot8}a^4\cos.^4\varphi + \&c.$$

and

$$\frac{1}{(1+a\cos.\varphi)^{\frac{3}{2}}} = 1 - \frac{3}{2}.a\cos.\varphi + \frac{3\cdot5}{2\cdot4}a^2\cos.^2\varphi - \frac{3\cdot5\cdot7}{2\cdot4\cdot6}a^3\cos.^3\varphi + \frac{3\cdot5\cdot7\cdot9}{2\cdot4\cdot6\cdot8}a^4\cos.^4\varphi + \&c.$$

$$\frac{1}{(1-a\cos.\varphi)^{\frac{3}{2}}} + \frac{1}{(1+a\cos.\varphi)^{\frac{3}{2}}} = 2 + \frac{3\cdot5}{4}a^2\cos.^2\varphi + \frac{3\cdot5\cdot7\cdot9}{4\cdot6\cdot8}a^4\cos.^4\varphi + \&c.$$

So that the equation (B) will become

$$M - F.\left\{\begin{array}{l} \frac{R-f}{\left\{(R-f)^2+r^2\right\}^{\frac{3}{2}}}.\left\{2 + 3.5.\frac{r^2(R-f)^2}{\left\{(R-f)^2+r^2\right\}^2}.\cos.^2\varphi\right\} \\ -\frac{R+f}{\left\{(R+f)^2+r^2\right\}^{\frac{3}{2}}}.\left\{2 + 3.5.\frac{r^2(R+f)^2}{\left\{(R+f)^2+r^2\right\}^2}.\cos.^2\varphi\right\} \end{array}\right\} = 0 \quad (C)$$

neglecting the terms which contain the fourth and higher powers of cos. φ,

Taking one of the cases which I investigated, and from which the others do not differ very considerably, the values of the co-efficients of cos. φ in the denominators of the fractions in the equation (B) are .25691 and .15951; so that the greatest of the terms neglected would be

$\frac{3\cdot5\cdot7\cdot9}{4\cdot6\cdot8}\times(.25691)^4.\cos.^4\varphi$ and $\frac{3\cdot5\cdot7\cdot9}{4\cdot6\cdot8}\times(.15951)^4\cos.^4\varphi$.

Now, supposing that φ is 70°, if these terms are employed in determining the value of F, it will be 218.7705 . M, and 218.8184 . M, if they are neglected; making a difference of .0479 M, or only affecting the fifth figure in this extreme

case. If, instead of expanding the fractions, we computed them in the form which they have in the equation (B), we could hardly be supposed to obtain the *absolute* values of F more nearly than this; although in either case the relative values would be obtained to a much greater degree of accuracy. In the observations which I made, the values of φ were seldom much less than 80°, and in such cases the error would be considerably less. In an instance where φ was 82° 37′, the value of F was 222.5630 M, employing the terms containing $\cos.^4 \varphi$, and 222.5640 M, neglecting them. Seeing then that no sensible error would arise from neglecting these terms, I have in all cases made use of the equation (C), for determining the values of F. I now proceed to the experiments which I made for this purpose.

On this occasion I made use of the same compass which I had already used in the greater part of the observations detailed in my former paper, and distinguished there as No. 1; the magnets were also the same that I had used with this compass. The length of the needle is very accurately 6 inches. In order to determine the distance between the points which I ought to consider as the poles of the needle, I fixed it at right angles to the meridian; and bringing another needle, freely suspended, near to it, I moved the centre of this needle along a line parallel to the axis of the first, and noted the points opposite to which the axis of the second was exactly in the magnetic meridian; these points I considered as the poles of the first needle. The distance between the points thus determined was 4.28 inches.

In my former paper I have stated the length of each of the magnets to be 12 inches; more accurately, the length of the

two joined together was 23.84 inches; so that the length of each might be taken to be very accurately 11.92 inches: they are .95 inch wide, and .375 inch thick. In all my observations the same magnet was always placed on the same side of the centre of the needle; so that in ascertaining the situations of their poles I distinguish one as the north, the other as the south magnet. The distances of the poles of the magnets from their ends, determined in the same manner as for the needle, were measured on each side, and a mean of the whole taken to obtain the distances between the poles; they were these:

	North Magnet.		South Magnet.	
	North Pole.	South Pole.	North Pole.	South Pole.
	0. 82 inch.	0.86 inch.	0.84 inch.	0. 77 inch.
	0. 81	0.86	0.84	0. 76
Mean	0.815	0.86	0.84	0.765

Taking half the sum of these, 1.64 inches, from the length of each magnet, we have 10.28 inches for the distance between the poles.

A meridian line being drawn on a firm table, standing on a stone floor, the compass was accurately adjusted on it, so that the needle pointed to zero on the graduated circle. The magnets were fixed at the bottoms of earthen pans, secured in such a way to rectangular pieces of board that their positions could not be accidentally changed, and projecting from these boards were small pieces of brass, on each of which a line was drawn to indicate the position of the axis of the magnet; the horizontal distance of the edge of each of the

projections nearest to the needle from the corresponding end of the magnet within the pan, was exactly 9 inches; I could therefore, in any instance, determine very accurately the distance of the centre of the magnet from that of the needle. The pans were placed on the table, so that the indexes on the pieces of brass coincided with the meridian line. Water was now poured into the pans, and the temperature of the magnets was varied by varying the temperature of the water. The temperature of each magnet was ascertained by a thermometer placed in the water, with its bulb resting on that pole of the magnet which was nearest to the centre of the needle. In my first observations I however made use of only one thermometer, which was moved, during them, from one magnet to the other. In Plate I. Fig. 1, an apparatus of the same nature, which I subsequently made use of, is represented. This differs from that employed in these experiments only in having the boxes containing the magnets made to slide on a ruler, whose axis being in the magnetic meridian, and the axes of the magnets adjusted in the boxes also in the meridian, they can be made to approach or recede from the needle, in that line, which saves considerable trouble in the adjustments when observations are to be made at different distances. Fig. 2 and 3 represent the plan and elevation of another apparatus which I had constructed for Mr. FOSTER, and which he has taken with him on the North-western Expedition, to enable him to make observations on the daily variation, particularly with a view of ascertaining the times of maximum east and west, and also of zero, should any of the stations at which he may find himself be favourable to the employment of such an ap-

Fig. 1.

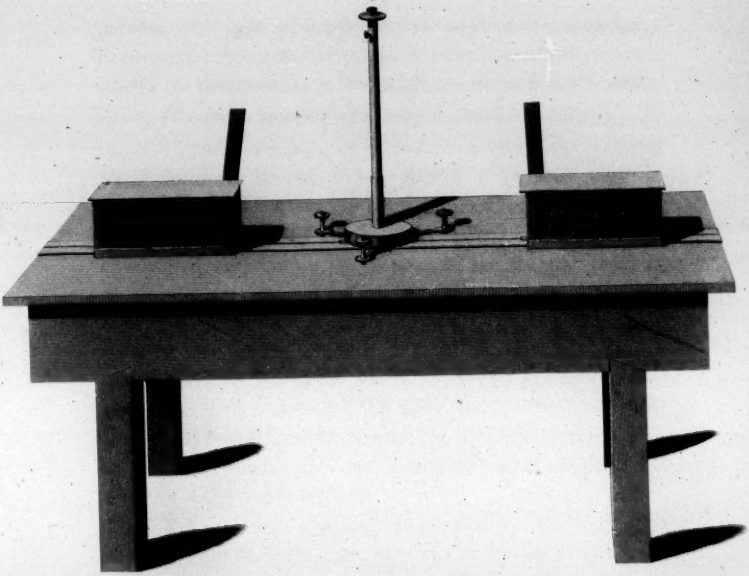


Fig. 2.

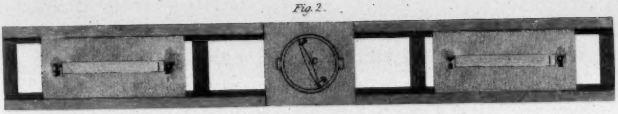


Fig. 3.

paratus. N, S, (Fig. 2,) denote the ends of the instrument to be placed towards the magnetic north and south; *c*, the centre of the needle; *n* and *s*, its north and south *poles*; ν_N, σ_N, the north and south *poles* of the north magnet; ν_S, σ_S, the north and south *poles* of the south magnet: the magnets being fixed on boards which, sliding in grooves, may be made to approach or recede from the needle at pleasure.

In the observations which I first made on the effects of changes of temperature, the centres of the magnets were at the same distances from the centre of the needle as they were during the observations on the diurnal changes in the directions of the needle, which it was my object to reduce, and which will be given in the conclusion of this paper: this distance was 21.21 inches. We have therefore in this case $R = 21.21$ inches, and, from what I have before said, $r = 2.14$ inches, and $\rho = 5.14$; if we substitute these values in the equation (C), it will become

$$M. - F. (.004690814 + .000829329 \cos. \phi) = 0. \qquad (\alpha)$$

Observing then the value of ϕ at any particular temperature of the magnets, by means of this equation I could readily obtain the corresponding value of F in terms of M; and by varying the temperature of the magnets, I obtained the variation of the intensity of their forces, corresponding to such change of temperature. The observations contained in the following table were made thus: I first noted the time, which is set down in the first column, and then the temperature of the north magnet; after which I placed the thermometer on the pole of the south magnet: I next observed the westerly point, at which the needle was held in equilibrio

by the terrestrial forces and those of the magnets, slightly agitating the needle, that it might the more readily assume the true position; from this it was led, by means of a very small and weak magnet, held on the outside of the compass-box, towards the easterly point of equilibrium, which was observed in the same manner; and from this it was led in the same way towards the southerly point, which, however, was not observed with an intention of deducing any thing by means of the equation (α), which was not calculated for such a value of ϕ. After these observations of the points of equilibrium, the temperature of the south magnet being observed, the time set down in the seventh column, at which the observations concluded, was noted. The temperature of the water in the pans was now increased or diminished, according to circumstances, by the addition of other water, and the pans covered over, to prevent any rapid changes of temperature during the observations: after allowing a short time for the magnets to acquire the temperature of the water, the observations were repeated. To prevent any ambiguity, with regard to the time indicated being morning or evening, I have, except when otherwise expressed, adopted the astronomical division of the day, from noon to noon. The scale made use of for the temperature was in all cases that of FAHRENHEIT.

Table of the positions of the Points of Equilibrium, corresponding to different Temperatures of the Magnets retaining a Magnetic Needle in equilibrio. 6th June, 1823.

Time of commencing observation.	Temperature of N. Magnet.	Points of Equilibrium. West.	East.	South.	Temperature of S. Magnet.	Time of concluding observation.	Mean temperature of the Magnets.	30.12 Barom. 60.25 Therm. attached
h. m.	°	° ′	° ′	° ′	°	h. m.	°	10h 15m
7 54	62.0	82 44	82 30	0 12W	62.1	8 01	62.05	
8 10	59.3	86 00	84 50	0 12	58.8	8 17	59.05	
8 31	79.0	74 40	74 56	0 12	76.3	8 35	77.65	
8 40	75.0	76 02	75 54	0 12	73.0	8 44	74.00	
8 52	71.0	77 30	77 24	0 10	70.3	8 56	70.65	
9 11	67.3	79 16	78 52	0 14	67.0	9 16	67.15	
9 34	63.8	80 42	80 50	0 18	63.8	9 40	63.80	
9 53	62.0	81 50	82 16	0 16	62.1	10 00	62.05	

Taking half the sum of the easterly and westerly arcs for the value of φ, and substituting them successively for φ in the equation (α), I obtain the values of $\frac{F}{M}$ corresponding to the respective mean temperatures of the magnets. These I have arranged in the following table; placing in the second column the differences of the successive temperatures, and in the fifth the corresponding differences in the values of $\frac{F}{M}$; these, divided by the numbers in the second column, will give the variation of the value of $\frac{F}{M}$, corresponding to a change in the temperature of the magnets of 1° on FAHRENHEIT's scale: these variations in the values of $\frac{F}{M}$ are contained in the last column of the table, and are denoted by $\Delta . \frac{F}{M}$.

Table of the Magnetic Intensities corresponding to different Temperatures of the Magnets.

Mean temperature of the Magnets.	Diff. of Temp. in successive observations.	Mean of the observed values of φ.	Magnetic Intensity or values of $\frac{F}{M}$.	Diff. of successive values of $\frac{F}{M}$.	Variation of $\frac{F}{M}$ for 1° Fah. or $\Delta \cdot \frac{F}{M}$.
°		° ′			
62.05		82 37	212.5620		
	− 3.00			+0.3803	0.1268
59.05		85 25	212.9423		
	+18.60			−2.3195	0.1247
77.65		74 48	210.6228		
	− 3.65			+0.3664	0.1004
74.00		75 58	210.9892		
	− 3.35			+0.4286	0.1279
70.65		77 27	211.4178		
	− 3.50			+0.4175	0.1193
67.15		79 04	211.8353		
	− 3.35			+0.3814	0.1138
63.80		80 46	212.2167		
	− 1.75			+0.2473	0.1413
62.05		82 03	212.4640		

The differences in the deduced values of the variation of $\frac{F}{M}$ for a change of temperature in the magnets of 1° in the last column, are not greater than we may suppose to have arisen from small inaccuracies in the observations, or slight changes in the terrestrial intensity during the time in which they were made; the latter indeed appear to have taken place, since, at the same temperature, the value of φ was 82° 37′ at the beginning of the observations, and 82° 03′ at their conclusion. The value 0.1247 deduced from the observations at the temperatures 59.05 and 77.65 I should consider as nearest the truth, since whatever may have been the errors, the divisor is here larger than in any other case; and, in taking a mean, this value should be taken with the mean of all the others: the contrary may be said of the value 0.1413, which should have only half the weight of any of the others. I therefore first take in this manner the mean of all the values excluding 0.1247, and then the mean of this

mean and 0.1247, and I thus get .1226 as the mean variation of the intensity of the magnets for a change in their temperature of 1°, between the temperatures 59.05 and 77.65, an increase of temperature always causing a decrease of intensity, and *vice versa*.

In the results in the last column of this table there are no marked indications of an increase in the values of $\Delta . \frac{F}{M}$ arising from an increase of the temperature at which the observations were made. Having afterwards, when I carried the observations to a greater extent in the scale of temperature, clearly ascertained that this was the case, I determined therefore not to take the mean of the values of $\Delta . \frac{F}{M}$, as I have here pointed out, but as I had made observations at every convenient opportunity, to take out from them, in the first place, all the values of $\Delta . \frac{F}{M}$, where the mean between the temperatures from which they were derived agreed nearly with the lowest temperature of the observations which it was my object to reduce; in the same manner, to take those which agreed most nearly with the mean temperature to which these observations were to be reduced; and likewise those agreeing with their highest temperature: taking then the mean of each of these, from these three means, I derived a value of $\Delta \frac{F}{M}$, from which I determined the variation of the angle ϕ, corresponding to any change of temperature. I have mentioned this here, that my reason for giving so many of the observations may be apparent. Observations, precisely similar to the preceding, were made on the 7th of June: they are contained in the following table.

Table of the positions of the Points of Equilibrium corresponding to different Temperatures of the Magnets retaining a Magnetic Needle in equilibrio, 7th June.

Time of commencing observation.	Temperature of N. Magnet.	Points of Equilibrium. West.	East.	South.	Temperature of S. Magnet.	Time of concluding observation.	Mean temperature of the Magnets.	30.12 Barom. 60.50 Therm. att.
h. m.	°	° ′	° ′	° ′	°	h. m.	°	
9 50	57.0	83 18	83 16	0 10W	57.0	9 53	57.00	
10 04	66.3	78 16	78 04	0 16	67.7	10 07	67.00	
10 15	71.0	76 22	76 10	0 14	70.7	10 18	70.85	
10 27	75.0	74 36	74 42	0 12	75.0	10 31	75.00	
10 44	60.0	81 18	80 26	0 20	61.0	10 47	60.50	10h 50m

Previous to making these observations I had slightly changed the distances of the magnets from the centre of the needle: the distances of their nearest ends were now 15.26 inches, or that of their centres 21.22 inches from the needle's centre. Substituting this value of R in the equation (C) it becomes

$$M - F\,(.004683954 + .000827265\cos.^2\phi) = 0; \qquad (\alpha_1)$$

and from this I calculated the following table.

Table of the Magnetic Intensities corresponding to different Temperatures of the Magnets.

Mean temperatures of the Magnets.	Diff. of Temp. in successive observations.	Mean of the observed values of ϕ.	Magnetic Intensity or value of $\frac{F}{M}$.	Diff. of successive values of $\frac{F}{M}$.	Variation of $\frac{F}{M}$ for 1° Fah. or $\Delta . \frac{F}{M}$.
°		° ′			
57.00		83 17	212.9803		
	+10.00			−1.0594	0.1059
67.00		78 10	211.9209		
	+ 3.85			−0.5302	0.1377
70.85		76 16	211.3907		
	+ 4.15			−0.5059	0.1219
75.00		74 39	210.8848		
	−14.50			+1.6641	0.1148
60.50		80 52	212.5489		

After these observations the magnets were wiped dry, and their poles of contrary names joined by bars of soft iron: to this circumstance I attribute the increase which, when next I used them, I found had taken place in their intensities. In the observations subsequent to these, I made use of two thermometers, one for each magnet, and observed the temperatures of both magnets at the beginning and at the conclusion of the observation.

Table of the positions of the Points of Equilibrium corresponding to different Temperatures of the Magnets retaining a Magnetic Needle in Equilibro. 13th of June.

Time of commencing observation.	Temperature of		Points of Equilibrium.			Temperature of		Time of concluding observation.	Mean Temperature of the Magnets.	
	N. Mag.	S. Mag.	West.	East.	South.	N. Mag.	S. Mag.			
h. m.	°	°	° ′	° ′	° ′	°	°	h. m.		
7 12	63.0	62.6	80 28	80 28	0 18 E	63.0	62.6	7 14	62.80	
7 35	61.1	61.0	81 12	81 16	0 16	61.2	61.0	7 40	61.08	
7 55	71.1	71.1	75 36	75 44	0 18	71.0	71.0	8 00	71.05	
8 20	66.2	65.8	77 40	77 48	0 18	66.1	65.7	8 32	65.95	8h 40m { Barom. 29.93 / Therm. att. 65.66

I have just mentioned that, on making these observations, I found the intensities of the magnets increased: on this account I was under the necessity of increasing their distances from the needle. The distances of their nearest ends from the centre of the needle were in this case 15.45 inches, or of their centres 21.41 inches: this value of R being substituted in the equation (C) gives

$$M - F\,(.004553604 + .0007880523\,\cos.^{4}\phi) = 0. \qquad (\alpha_2).$$

As before, I calculate the following table from this equation.

Table of the Magnetic Intensities corresponding to different Temperatures of the Magnets.

Mean Temperature of the Magnets.	Diff. of Temp. in successive observations.	Mean of the observed values of φ.	Magnetic Intensity or value of $\frac{F}{M}$.	Diff. of successive values of $\frac{F}{M}$.	Variation of $\frac{F}{M}$ for 1° Fah. or $\Delta \cdot \frac{F}{M}$.
°		° ′			
62.80		80 28	218.5687		
	−1.72			+0.1582	0.0920
61.08		81 14	218.7269		
	+9.97			+1.4255	0.1430
71.05		75 40	217.3014		
	−5.10			+0.6026	0.1182
65.95		77 44	217.9040		

There is only one, the first, of the values of $\Delta \cdot \frac{F}{M}$, which differs much from those already obtained, but the difference of the temperatures in the observations from which it is derived is so small, that any errors would be rendered very sensible; and if the thermometers happened not to indicate the precise temperatures of the magnets at the times of observation, it would be quite sufficient to account for this discrepancy.

In his paper on the daily variation of the horizontal and dipping needles under a reduced directive power, Mr. BARLOW has described some anomalies which he observed between the daily changes in the direction of a needle when placed in the house and when in the open air, and also the steps which he took to discover their cause. He mentions, "that in certain positions of the needle towards the east and west, the daily motion, although it proceeded with the same determinate uniformity in both cases, yet it took place in different directions; passing in the one instance from the east, or west, towards the south, and in the other towards the north, at the

same corresponding hours of the day, the motion in both instances being equally distinct, regular, and progressive."* These anomalies, I also noticed, although, as I have mentionod in my former paper, I did not find the reversion, in the directions in the two cases, to take place with the same regularity and uniformity that Mr. BARLOW observed it to have. In that paper I also stated my opinion, that these anomalies had arisen from the difference in the changes of temperature in the magnets when in doors and when in the open air, and that the observations in the two cases would be found to agree when they were freed from the influence of difference of temperature in the magnets.

As I had already made observations in doors, in which I noted the temperature of the magnets, it was now my intention to make corresponding observations in the open air, in order that by reducing the obervations to the same standard of temperature, their agreement or disagreement might be put beyond doubt. For this purpose the whole apparatus was placed in my garden, exposed to the sun and air, on a table having its legs driven firmly into the ground; and for several days I observed, at stated intervals, the positions of the points of equilibrium; when I had an opportunity I also made experiments, similar to the preceding, for the purpose of determining the value of $\Delta \frac{F}{M}$, to be applied to the correction of the observations in doors and in the open air.

On adjusting the magnets to the needle, I again found that

* In the Postscript to this paper, Mr. BARLOW, to whom I had communicated my views with regard to the effects of temperature, refers to the experiments which I had made, for the explanation of these apparent anomalies.

their intensities had increased, owing, I consider, to the same circumstance as before, and I therefore increased their distances from the needle; but after making the first days observations, and comparing them with those made in doors, I found it necessary slightly to diminish these distances, in order that, at the same temperature of the magnets, the situations of the points of equilibrium might more nearly agree in the two cases. During the observations of the first day, the distance of the nearest ends of the magnets from the centre of the needle were 15.62 inches: so that the value of R is here 21.58 inches, and the equation C becomes,

$$M - F\,(.004441190 + .0007549085\,\cos.^2\varphi) = 0 \qquad (\alpha_3)$$

The observations are contained in the following table.

Table of the positions of the Points of Equilibrium corresponding to different Temperatures of the Magnets retaining a Magnetic Needle in Equilibrio.
17th and 18th June.

Time of commencing observation.		Temperature of the Magnets.		Points of Equilibrium.				Temperature of the Magnets.		Time of concluding observation.	Mean Temperature of the Magnets.	Barom.	Therm. attached.
		North.	South.	West.	East.	South.		North.	South.				
June	h. m	°	°	° ′	° ′	° ′		°	°	h. m.	°		
17	19 27	49.6	48.8	79 58	80 46	0 42	E	49.8	49.0	19 32	49.30		
	19 53	60.25	60.25	75 24	75 46	0 32	E	60.25	60.25	19 56	60.25		
	20 15	68.4	68.6	72 02	72 40	0 24	E	67.8	68.2	20 19	68.25		
	20 36	74.9	75.0	69 36	70 08	0 24	E	74.2	74.3	20 41	74.60		
	21 02	61.8	62.0	74 12	74 24	0 14	E	61.5	61.7	21 06	61.75		
	21 26	74.0	74.2	70 06	70 24	0 08	E	73.4	73.6	21 30	73.80	30.29	55.75
18	7 36	55.7	55.5	75 00	75 06	0 00		55.7	55.4	7 40	55.58		
	8 00	66.2	66.0	70 58	71 28	0 02	E	66.0	65.8	8 04	66.00		
	8 25	73.8	73.8	68 12	68 52	0 02	E	73.4	73.4	8 28	73.60		
	8 51	56.8	57.4	74 38	74 44	0 02	E	56.4	57.0	8 56	56.90	30.20	56.00

From these I calculate the following table by means of the equation (α_3).

Table of the Magnetic Intensities corresponding to different Temperatures of the Magnets.

Mean Temperature of the Magnets.	Diff. of Temp. in successive observations.	Mean of the observed values of φ.	Magnetic Intensity, or value of $\frac{F}{M}$.	Diff. of successive values of $\frac{F}{M}$.	Variation of $\frac{F}{M}$ for 1° Fah or $\Delta . \frac{F}{M}$.
°		° ′			
49.30		80 22	224.0981		
60.25	+10.95	75 35	222.8171	−1.2810	0.1179
68.25	+ 8.00	72 21	221.7046	−1.1125	0.1391
74.60	+ 6.35	69 52	220.7198	−0.9848	0.1551
61.75	−12.85	74 18	222.3967	+1.6769	0.1305
73.80	+12.05	70 15	220.8778	−1.5189	0.1260
55.58		75 03	222.6462		
66.00	+10.42	71 13	221.2655	−1.3807	0.1315
73.60	+ 7.60	68 32	220.1532	−1.1123	0.1461
56.90	−16.70	74 39	222.5145	−2.3613	0.1314

After making these obsevations, the distances of the magnets from the needle were slightly diminished, for the reasons I have already mentioned: their nearest ends were now 15.56 inches from the centre of the needle, or the value of R was 21.52 inches. By substituting this value in the equation C, it becomes

$$M - F . (.00448043 + .0007664093 \cos.^2\varphi) = 0. \qquad (\alpha_4)$$

The observations which I made with the magnets at this distance from the needle, and the results which I obtain from them, are contained in the two following tables.

Table of the positions of the Points of Equilibrium corresponding to different temperatures of the Magnets retaining a Magnetic Needle in equilibrio. 18th, 19th, 20th, 22nd June.

Time of commencing observation.		Temperature of the Magnets.		Points of Equilibrium.				Temperature of the Magnets.		Time of concluding observation.	Mean Temperature of the Magnets.	Barom.	Therm. attached.
		North.	South.	West.		East.	South.	North.	South.				
June	h. m.	°	°	° ′		° ′	°	°	°	h. m.	°		°
18	19 24	55.2	55.2	85	18	84 24	0 20 E	55.6	55.6	19 32	55.40	30.23	57.10
	19 47	74.0	74.2	75	16	74 24	0 26 E	73.5	73.5	19 51	73.80		
	20 08	55.5	55.3	85	36	84 32	0 14 E	55.5	55.3	20 14	55.40		
19	7 25	56.0	55.0	83	08	82 06	0 06 E	55.8	55.0	7 30	55.45		
	7 52	64.6	64.3	77	38	77 20	0 00	64.3	64.0	7 56	64.30	30.17	55.3
	8 16	74.4	74.2	73	28	72 58	0 00	73.5	73.5	8 21	73.90		
	8 40	64.7	65.0	77	44	77 06	0 12W	63.7	63.7	8 47	64.28		
	9 01	55.6	55.8	82	34	81 30	0 12W	55.3	55.5	9 04	55.55	30.17	55.7
	19 25	55.2	54.8	84	12	83 02	0 18 E	55.5	55.1	19 29	55.15	30.11	56.75
	19 50	67.3	66.5	77	26	76 44	0 22 E	66.8	66.4	19 55	66.75		
20	17 29	51.3	50.0	85	16	85 14	0 14 E	51.3	50.0	17 34	50.65	30.10	60.4
	17 50	56.8	55.0	81	46	81 28	0 26 E	56.6	54.8	17 55	55.80		
	18 14	51.7	51.1	84	44	84 38	0 16 E	51.6	51.1	18 19	51.35		
	18 40	57.6	57.0	80	42	80 46	0 20 E	57.4	56.8	18 46	57.20		
22	18 01	54.7	53.7	82	52	82 46	0 10 E	54.5	53.4	18 06	54.08	30.16	55.3
	18 22	52.0	51.0	85	10	85 14	0 10 E	51.6	50.8	18 28	51.35		
	18 48	56.3	56.0	82	08	81 40	0 14 E	55.8	55.7	18 53	55.94		

Table of the Magnetic Intensities corresponding to different Temperatures of the Magnets.

Mean Temperature of the Magnets.	Diff. of Temp. in successive observations.	Mean of the observed values of ϕ.	Magnetic intensity or value of $\frac{F}{M}$.	Diff. of successive values of $\frac{F}{M}$.	Variation of $\frac{F}{M}$ for 1° Fah. or $\Delta . \frac{F}{M}$.
°		° ′			
55.40		84 51	222.8859		
	+18.40			−2.2756	0.1237
73.80	−18.40	74 50	220.6103	+2.3010	0.1251
55.40		85 04	222.9113		
55.45	+ 8.85	82 37	222.5640	−1.1496	0.1299
64.30	+ 9.60	77 29	221.4144	−1.3595	0.1415
73.90	− 9.62	73 13	220.0549	+1.3413	0.1394
64.28	− 9.73	77 25	221.3962	+1.0648	0.1220
55.55		82 02	222.4610		
55.15	+11.60	83 37	222.7217	−1.4204	0.1224
66.75		77 05	221.3013		
50.65	+ 5.15	85 15	222.9322	−0.5474	0.1063
55.80	− 4.45	81 37	222.3848	+0.4712	0.1059
51.35	+ 5.85	84 41	222.8660	−0.6580	0.1125
57.20		80 44	222.2080		
54.08	− 2.73	82 49	222.5974	+0.3289	0.1203
51.35	+ 4.59	85 12	222.9263	−0.4886	0.1064
55.94		81 54	222.4377		

After having made the preceding observations, and concluded those on the diurnal changes in the points of equilibrium, I proposed applying the balance of torsion, as another means of determining the variations in the magnetic intensities, arising from changes in the temperatures of the magnets; but in my first observations with this instrument, there were such discrepancies, arising from the over torsion of the wire in consequence of its want of elasticity, being silver and too fine for the weight of the needle, and likewise too short, that, although they pointed out very clearly the same general results which I afterwards obtained from unexceptionable observations, I would not make use of them for determining a mean value of $\Delta . \frac{F}{M}$ to be applied to the correction of the observations on the diurnal changes, for the variation in the temperature of the magnets. As a considerable time intervened before I had an opportunity of repeating these experiments, and in making them, I had, by increasing the temperature of the magnets beyond a certain point, permanently destroyed a portion of their intensities, I considered it better to obtain the mean value of $\Delta . \frac{F}{M}$, which was requisite, from the results of the experiments made more nearly at the same time as the observations which it was my object to reduce. I therefore determined this value from the preceding results.

The temperatures of the magnets in the observations on the daily changes of the points of equilibrium were, with a few exceptions above and below, comprised between 54° and 65°, and I determined the mean value of $\Delta \frac{F}{M}$ to be applied to the correction of these observations in this manner: from

the preceding results I first took those values of $\Delta \frac{F}{M}$ derived from observations where the mean of the temperatures was near to 65°, and from these obtained a mean value of $\Delta . \frac{F}{M}$ when the mean temperature of the magnets was nearly 65°: in like manner I obtained a mean value of $\Delta . \frac{F}{M}$ when the mean temperature of the magnets was nearly 60°, and also when nearly 54°: taking a mean of these three means, I obtained a value of $\Delta . \frac{F}{M}$, which could not be sensibly different from its true value in any of the observations which were to be corrected. These are collected in the following table.

Table of Results for obtaining the Mean value of $\Delta . \frac{F}{M}$.

Mean Temperature 65° nearly.					Mean Temperature 60° nearly.					Mean Temperature 54° nearly.				
Date.	Temperatures.		Mean.	Value of $\Delta . \frac{F}{M}$	Date.	Temperatures.		Mean.	Value of $\Delta . \frac{F}{M}$	Date.	Temperatures.		Mean.	Value of $\Delta . \frac{F}{M}$
June	°	°	°		June	°	°	°		June	°	°	°	
6	62.05	67.15	64.600	.1233	6	59.05	62.05	60.550	.1268	17	49.30	60.25	54.775	.1179
13	61.08	71.05	66.065	.1430	17	49.30	68.25	58.775	.1263	20	50.65	55.80	53.225	.1063
17	60.25	68.25	64.250	.1391	18	55.58	66.00	60.790	.1315	20	51.35	55.80	53.575	.1059
17	61.75	73.80	67.775	.1260	19	55.45	64.30	59.875	.1299	20	51.35	57.20	54.275	.1125
18	56.90	73.60	65.250	.1314	19	55.55	64.28	59.915	.1220	22	51.35	54.08	52.715	.1203
18	55.40	73.80	64.600	.1237	19	55.15	66.75	60.950	.1224	22	51.35	55.94	53.645	.1064
18	55.40	73.80	64.600	.1251			Mean	60.143	.12648			Mean	53.701	.11155
19	55.45	73.90	64.675	.1360										
19	55.55	73.90	64.725	.1311										
		Mean	65.171	.13097	Mean value of $\Delta . \frac{F}{M}$, to be applied in correcting the observations on the Diurnal changes in the positions of the Points of Equilibrium				0.12300					

To apply the value of $\Delta . \frac{F}{M}$, thus determined, to the correction of the observed directions of the needle, for the changes which took place in the temperature of the magnets, let $\Delta \phi$ be the increment of the azimuth of the needle corres-

ponding to the increment ΔF of the intensity of the magnets. When therefore F becomes $F + \Delta F$, putting the equation (C) in the form

$$M - (P + Q \cdot \cos.^{n} \varphi) \cdot F = o,$$

it becomes

$$M - \left\{P + Q \cdot \cos.^{n} (\varphi + \Delta \varphi)\right\} (F + \Delta F) = o,$$

whence

$$\cos.^{n} (\varphi + \Delta \varphi) = \frac{1}{Q} \cdot \left\{\frac{M}{F + \Delta F} - P\right\}. \qquad (E)$$

This formula, though sufficiently simple, is not in the most convenient form for calculating the values of $\Delta \varphi$, corresponding to different values of φ, owing to the common tables of logarithms giving only seven places of figures. The values of P and $\frac{M}{F + \Delta F}$ agree in the first two or three figures, so that there will remain only five figures towards the beginning of the table, and four figures towards the end of it, from which the values of $\varphi + \Delta \varphi$ are to be derived, consequently they cannot be calculated to the greatest accuracy. But if $\frac{M}{F + \Delta F}$ be expanded, the first figure of $\frac{M}{F + \Delta F} - P$ being in the 5th place of decimals, the first figure of $\frac{(\Delta F)^{2}}{F^{3}}$ will be in the 9th place, and the first figure of $\frac{(\Delta F)^{3}}{F^{4}}$ will be in the 13th place; and therefore we should obtain the value of $\frac{M}{F + \Delta F} - P$ true to the 11th place of decimals, or true to 7 places of figures when we neglect the term $\frac{(\Delta F)^{2}}{F^{3}}$. Now in the cases which I had to compute, the first two figures in the value of $\frac{(\Delta F)^{2}}{F^{3}}$ were the same for all the arcs, and conse-

quently by using these, the value of $\frac{M}{F+\Delta F} - P$ for the determination of $\cos.^2(\varphi + \Delta\varphi)$ would be true to the 6th figure, which would give $\varphi + \Delta\varphi$ to the tenth of a second. Such a degree of accuracy may appear quite uncalled for by the nature of the observations, but from the manner which I adopted for correcting them, it was necessary to guard against any accumulation of error.

From what I have said, we have

$$\cos.^2(\varphi + \Delta\varphi) = \frac{1}{Q}\left\{\frac{M}{F} - M \cdot \left(\frac{\Delta F}{F^2} - \frac{(\Delta F)^2}{F^3}\right) - P\right\};$$

but

$$\frac{M}{F} = P + Q\cos.^2\varphi,$$

and therefore,

$$\cos.^2(\varphi + \Delta\varphi) = \frac{1}{Q}\left\{Q\cos.^2\varphi + \frac{M(\Delta F)^2}{F^3} - \frac{M.\Delta F}{F^2}\right\}. \qquad (G)$$

Having, as we have seen, determined by observation ΔF in terms of M, and $\frac{M}{F}$, being computed from the equation (C), for any angle φ, the value of $\Delta\varphi$ would be readily computed from this formula: that is, we could obtain from it the correction to be made in any observed angle, for a change of 1° in the temperature of the magnets, whether that temperature were increasing or decreasing, only observing that ΔF is minus for an increase of temperature, and plus for a decrease.

The method which I have adopted for reducing the observed values of φ to what they would have been, had the temperature of the magnets been constant, is this: the observed values of φ being comprised between 74° and 86°, I computed the values of $\Delta\varphi$, both plus and minus, at inter-

vals of 30 minutes, from 74° to 86°, by means of the formula (G): from these and their several orders of differences, I interpolated the values of $\Delta\phi$ at intervals of 6 minutes: forming these, with their differences, into tables, I obtained from them, by inspection, the value of $\Delta\phi$ corresponding to any observed angle: adding the plus value of $\Delta\phi$ to the observed angle, when the temperature of the magnets was above the mean temperature to which the observations were to be reduced, I obtained the value of ϕ at a temperature of the magnets one degree lower than that observed: proceeding in the same manner with this corrected value of ϕ, I obtained another value at a temperature one degree lower than the last, or two degrees below the observed temperature: with this I proceeded again in the same manner, and so on, until the observed value of ϕ was reduced to its value at the standard temperature of the magnets. If the observed temperature was below the mean temperature, I successively subtracted the different minus values of $\Delta\phi$ to obtain the corrected value of ϕ. This will perhaps be better understood when I come to the observations and their corrections; but I thought it necessary to explain the use which I made of these tables previous to giving them.

In the observations which I made within doors on the daily variation in the positions of the points of equilibrium, the distances of the nearest ends of the magnets from the centre of the needle were 15.21 inches, or the distances of their centres from the centre of the needle 21.21 inches; so that, as we have before seen, the equation (C) here becomes

$$M - (.004690814 + .000829329 \cos.^2\Phi).F = 0; \quad (\alpha)$$

consequently the equation (G) becomes

$$\cos.^2(\varphi+\Delta\varphi)=\frac{1}{.000829329}\times\left\{.000829329\cos.^2\varphi+.0000000016\right.$$
$$\left.\mp.123\times(.004690814+.000829329\cos.^2\varphi)^2\right\}=0;\ldots.(\gamma)$$

.0000000016, being the value of $\frac{M.(\Delta F)^2}{F^2}$ in all the values of φ for which I had to compute; and .123 the value of $\Delta.\frac{F}{M}$ already found: the upper sign to be used when $\Delta.\frac{F}{M}$ is plus, or when the observed temperature of the magnets is above the mean temperature to which the observations are to be reduced, and the lower sign, when $\Delta.\frac{F}{M}$ is minus.

This formula is not so ill adapted for calculation as it may at first sight appear, since for each value of φ it is only necessary to refer to the tables eight times to obtain the values both of $\varphi+\Delta\varphi$ and $\varphi-\Delta_1\varphi$, or of φ_1 and $\underline{\varphi}_1$.

The values of φ, in the observations in doors, being comprised between 77° and 86°, I calculated the two following tables as the basis of the tables by which these observations were to be corrected, for the difference between the observed temperature of the magnets and the standard temperature.

1. ***Table of the*** increments ***in the Azimuths of the Points of Equilibrium corresponding to a*** decrement ***of*** 1° ***in the Temperature of the Magnets, with their several orders of differences, calculated at intervals of*** 30′ ***in the Azimuths from*** 77° ***to*** 86°: ***the distances of the centres of the Magnets from the centre of the Needle being*** 21.21 ***inches.***

φ	Δ φ	Δ² φ	Δ³ φ	Δ⁴ φ	Δ⁵ φ	Δ⁶ φ	Δ⁷ φ
° ′	′	′	′	′	′	′	′
77 00	26.455	0.988					
30	27.443	1.080	0.092	0.012	0.004		
78 00	28.523	1.184	0.104	0.016	0.004		
30	29.707	1.304	0.120	0.020	0.004		
79 00	31.011	1.444	0.140	0.024	0.003		
30	32.455	1.608	0.164	0.027	0.008		
80 00	34.063	1.799	0.191	0.035	0.012		
30	35.862	2.025	0.226	0.047	0.017		
81 00	37.887	2.298	0.273	0.064	0.016	0.013	
30	40.185	2.635	0.337	0.080	0.029	0.021	
82 00	42.820	3.052	0.417	0.109	0.050	0.025	0.032
30	45.872	3.578	0.526	0.159	0.075	0.057	0.061
83 00	49.450	4.263	0.685	0.234	0.132	0.118	0.210
30	53.713	5.182	0.919	0.366	0.250	0.328	0.651
84 00	58.895	6.467	1.285	0.616	0.578	0.979	
30	65.362	8.368	1.901	1.194	1.557		
85 00	73.730	11.463	3.095	2.751			
30	85.193	17.309	5.846				
86 00	102.502						

2. *Table of the* decrements *in the Azimuths of the Points of Equilibrium corresponding to an* increment *of* 1° *in the Temperature of the Magnets, with their several orders of differences, calculated at intervals of* 30′ *in the Azimuths from* 77° *to* 86°: *the distances of the centres of the Magnets from the centre of the Needle being* 21.21 *inches.*

φ	$\Delta\varphi$	$\Delta^2\varphi$	$\Delta^3\varphi$	$\Delta^4\varphi$	$\Delta^5\varphi$
° ′	′	′	′	′	′
77 00	25.675	0.890	0.073	0.011	0.000
30	26.565	0.963	0.084	0.011	0.002
78 00	27.528	1.047	0.095	0.013	0.001
30	28.575	1.142	0.108	0.014	0.002
79 00	29.717	1.250	0.122	0.016	0.004
30	30.967	1.372	0.138	0.020	0.004
80 00	32.339	1.510	0.158	0.024	0.007
30	33.849	1.668	0.182	0.031	0.008
81 00	35.517	1.850	0.213	0.039	0.004
30	37.367	2.063	0.252	0.043	0.010
82 00	39.430	2.315	0.295	0.053	0.018
30	41.745	2.610	0.348	0.071	0.018
83 00	44.355	2.958	0.419	0.089	0.020
30	47.313	3.377	0.508	0.109	0.027
84 00	50.690	3.885	0.617	0.136	0.046
30	54.575	4.502	0.753	0.182	
85 00	59.077	5.255	0.935		
30	64.332	6.190			
86 00	70.522				

By means of the values of $\Delta\Phi$ and their several orders of differences, contained in these tables, interpolating in the usual manner, I calculated the following tables.

I. *Table of the* increments *in the Azimuths of the Points of Equilibrium corresponding to a* decrement *of* 1° *in the Temperature of the Magnets, calculated at intervals of* 6′ *in the Azimuths from* 77° *to* 86°; *the distances of the centres of the magnets from the centre of the needle being* 21.21 *inches: to be applied to the correction of the observed Azimuths, when the Observed Temperature of the Magnets is* above *the Mean Temperature to which the observations are to be reduced.*

φ	77°		78°		79°		80°		81°	
	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.
′	′		′		′		′		′	
00	26.455	.191	28.523	.228	31.011	.277	34.063	.343	37.887	.436
06	.646	.194	.751	.232	.288	.283	.406	.351	38.323	.447
12	.840	.197	.983	.237	.571	.288	.757	.360	.770	.459
18	27.037	.201	29.220	.241	.859	.295	35.117	.368	39.229	.471
24	.238	.205	.461	.246	32.154	.301	.485	.377	.700	.485
30	.443	.209	.707	.250	.455	.308	.862	.386	40.185	.498
36	.652	.212	.957	.255	.763	.315	36.248	.395	.683	.512
42	.864	.216	30.212	.261	33.078	.322	.643	.405	41.195	.527
48	28.080	.220	.473	.266	.400	.328	37.048	.414	.722	.541
54	.300	.223	.739	.272	.728	.335	.462	.425	42.263	.557
60	.523		31.011		34.063		.887		.820	

φ	82°		83°		84°		85°	
	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.
′	′		′		′		′	
00	42.820	.574	49.450	.791	58.895	1.173	73.730	1.985
06	43.394	.591	50.241	.820	60.068	1.228	75.715	2.120
12	43.985	.609	51.061	.851	61.296	1.288	77.835	2.272
18	44.594	.629	51.912	.883	62.584	1.354	80.107	2.445
24	45.223	.649	52.795	.918	63.938	1.424	82.552	2.641
30	45.872	.670	53.713	.955	65.362	1.499	85.193	2.865
36	46.542	.691	54.668	.994	66.861	1.579	88.058	.120
42	47.233	.714	55.662	1.034	68.440	1.665	91.178	3.411
48	47.947	.739	56.696	1.077	70.105	1.759	94.589	3.753
54	48.686	.764	57.773	1.122	71.864	1.866	98.342	4.160
60	49.450		58.895		73.730		102.502	

II. *Table of the* decrements *in the Azimuths of the Points of Equilibrium corresponding to an* increment *of* 1° *in the Temperature of the Magnets, calculated at intervals of* 6′ *in the Azimuths from* 77° *to* 86°; *the distances of the centres of the Magnets from the centre of the Needle being* 21.21 *inches: to be applied to the correction of the observed Azimuths when the observed Temperature of the Magnets is* below *the Mean Temperature to which the observations are to be reduced.*

φ	77°		78°		79°		80°		81°	
	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.
′	′		′		′		′		′	
00	25.675	.173	27.528	.203	29.717	.241	32.339	.290	35.517	.354
06	.848	.175	.731	.206	.958	.245	.629	.296	.871	.362
12	26.023	.178	.937	.209	30.203	.250	.925	.301	36.233	.370
18	.201	.181	28.146	.213	.453	.255	33.226	.308	.603	.378
24	.382	.183	.359	.216	.708	.259	.534	.315	.981	.386
30	.565	.187	.575	.220	.967	.264	.849	.320	37.367	.394
36	.752	.189	.795	.224	31.231	.269	34.169	.327	.761	.403
42	.941	.192	29.019	.228	.500	.274	.496	.333	38.164	.413
48	27.133	.196	.247	.233	.774	.280	.829	.340	.577	.422
54	.329	.199	.480	.237	32.054	.285	35.169	.348	.999	.431
60	.528		.717		.339		.517		39.430	

φ	82°		83°		84°		85°	
	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.
′	′		′		′		′	
00	39.430	.442	44.355	.562	50.690	.733	59.077	.985
06	.872	.452	.917	.576	51.423	.755	60.062	1.017
12	40.324	.462	45.493	.591	52.178	.776	61.079	1.049
18	.786	.474	46.084	.607	52.954	.799	62.128	1.084
24	41.260	.485	.691	.622	53.753	.822	63.212	1.120
30	.745	.497	47.313	.639	54.575	.847	64.332	1.157
36	42.242	.509	.952	.657	55.422	.873	65.489	1.195
42	.751	.522	48.609	.675	56.295	.899	66.684	1.236
48	43.273	.534	49.284	.693	57.194	.927	67.920	1.279
54	.807	.548	.977	.713	58.121	.956	69.199	1.323
60	44.355		50.690		59.077		70.522	

These tables are calculated for the distance 21.21 inches, that at which the centres of the magnets were from the centre of the needle during the observations which I made in-doors, and they would, without any great error, serve also for the correction of the observations in the open air, where the distances were 21.52 inches; but I would not, for the sake of avoiding the labour of computing fresh tables, which however was by no means inconsiderable, leave any doubt on the nature of the diurnal changes in the two cases.

We have already seen that, when $R = 21.52$, the equation (C) becomes,

$$M - F.(.004480432 + .0007664093 \cos.^3\varphi) = 0, \quad (\alpha_4)$$

so that in this case the equation G becomes

$$\cos.^3(\varphi + \Delta\varphi) = \frac{1}{.0007664093}\Big\} \times .0007664093 \cos.^3\varphi + .0000000014$$
$$\mp .123 \times (.004480432 + .0007664093 \cos.^3\varphi)^3\Big\} = 0 \quad (\gamma_1)$$

where .0000000014 is the value of $\frac{M.(\Delta F)^3}{F^3}$ in all the values of φ between 74° and 86°, the values which I had in this case to compute.

From this formula I calculated the following tables, as in the preceding case, excepting that, as in the observations in the open air, the temperature of the magnets varied more considerably, I had, in correcting them, more frequently to repeat the additions and subtractions, and therefore from 82° to 86°, where the values of $\Delta\varphi$ change rapidly, I calculated the values of $\Delta\varphi$ at intervals of 15′ for the fundamental tables, and interpolated at intervals of 3′ for the tables to be applied to the correction of the observations.

3. *Table of the* increments *in the Azimuths of the Points of Equilibrium corresponding to a* decrement *of* 1° *in the Temperature of the Magnets, with their several orders of differences, calculated at intervals of* 30′ *in the Azimuths from* 74° *to* 82°, *and at intervals of* 15′ *in those from* 82° *to* 86°; *the distances of the centres of the Magnets from the centre of the Needle being* 21.52 *inches.*

φ	Δ φ	Δ² φ	Δ³ φ	Δ⁴ φ	Δ⁵ φ	Δ⁶ φ	Δ⁷ φ
° ′	′	′	′	′	′	′	′
74 00	21.653	0.607	0.046				
30	22.260	0.653	0.051	0.005			
75 00	22.913	0.704	0.057	0.006			
30	23.617	0.761	0.063	0.005			
76 00	24.378	0.824	0.071	0.008	0.001		
30	25.202	0.895	0.080	0.009	0.002		
77 00	26.097	0.975	0.091	0.011	0.002		
30	27.072	1.066	0.103	0.012	0.003		
78 00	28.138	1.169	0.118	0.015	0.004		
30	29.307	1.287	0.137	0.019	0.005		
79 00	30.594	1.424	0.161	0.024	0.004	′	
30	32.018	1.585	0.189	0.028	0.007	0.003	
80 00	33.603	1.774	0.224	0.035	0.010	0.003	
30	35.377	1.998	0.269	0.045	0.015	0.005	
81 00	37.375	2.267	0.329	0.060			
30	39.642	2.596					
82 00	42.238	1.446	0.114				
15	43.684	1.560	0.129	0.015	0.003		
30	45.244	1.689	0.147	0.018	0.002		
45	46.933	1.836	0.167	0.020	0.005	0.001	
83 00	48.769	2.003	0.192	0.025	0.006	0.002	
15	50.772	2.195	0.223	0. 31	0.008	0.001	
30	52.967	2.418	0.262	0.039	0.009	0.007	
45	55.385	2.680	0.310	0.048	0.016	0.004	
84 00	58.065	2.990	0.374	0.064	0.020	0.010	0.005
15	61.055	3.364	0.458	0.084	0.030	0.015	0.018
0	64.419	3.822	0.572	0.114	0.045	0. 33	0.028
45	68.241	4.394	0.731	0.159	0.078	0.061	0.068
85 00	72.635	5.125	0.968	0.237	0.139	0.129	
15	77.760	6.093	1.344	0.376	0.268		
30	83.853	7.437	1.988	0.644			
45	91.290	9.425					
86 00	100.715						

4. ***Table of the*** **decrements** ***in the Azimuths of the Points of Equilibrium corresponding to an*** **increment** ***of*** 1° ***in the Temperature of the Magnets, with their several orders of differences, calculated at intervals of*** 30′ ***in the Azimuths from*** 74° ***to*** 82°, ***and at intervals of*** 15′ ***in those from*** 82° ***to*** 86° ; ***the distances of the centres of the Magnets from the centre of the needle being*** 21.52 ***inches.***

φ	Δφ	Δ²φ	Δ³φ	Δ⁴φ
° ′	′	′	′	′
74 00	21.249	0.566	0.040	0.004
30	21.815	0.606	0.044	0.005
75 00	22.421	0.650	0.049	0 006
30	23.071	0.699	0.055	0.005
76 00	23.770	0.754	0.060	0.005
30	24.524	0.814	0.065	0.008
77 00	25.338	0.879	0.073	0.010
30	26.217	0.952	0.083	0.012
78 00	27.169	1.035	0.095	0.010
30	28.204	1.130	0.105	0.014
79 00	29.334	1.235	0.119	0.018
30	30.569	1.354	0.137	0.021
80 00	31.923	1.491	0.158	0.024
30	33.414	1.649	0.182	0.029
81 30	35.063	1.831	0.211	
30	36.894	2.042		
82 00	38.936	1.111	0.066	
15	40.047	1.177	0.073	0.007
30	41.224	1.250	0.081	0.008
45	42.474	1.331	0.086	0.005
83 00	43.805	1.417	0.094	0.008
15	45.222	1.511	0.103	0.009
30	46.733	1.614	0.114	0.011
45	48.347	1.728	0.125	0.011
84 00	50.075	1.853	0.137	0.012
15	51.928	1.990	0.152	0.015
30	53.918	2.142	0.169	0.017
45	56.060	2.311	0.189	0.020
85 00	58.371	2.500	0.208	0.019
15	60.871	2.708	0.231	0.023
30	63.579	2.939	0.260	0.029
45	66.518	3.199		
86 00	69.717			

From these tables, interpolating as before, I constructed the two following.

III. *Table of the* increments *in the Azimuths of the Points of Equilibrium corresponding to a* decrement *of* 1° *in the Temperature of the Magnets, calculated at intervals of* 6′ *in the Azimuths from from* 74 *to* 82°, *and of* 3′ *in those from* 82° *to* 86°; *the distances of the centres of the Magnets from the centre of the Needle being* 21.52 *inches: to be applied to the correction of the observed Azimuths when the Observed Temperature of the Magnets is* above *the Mean Temperature to which the observations are to be reduced.*

φ	74°		75°		76°		77°	
′	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.
00	21.653	.118	22.913	.137	24.378	.160	26.097	.188
06	.771	.120	23.050	.138	.538	.162	.285	.192
12	.891	.121	.188	.141	.700	.164	.477	.195
18	22.012	.123	.329	.143	.864	.168	.672	.198
24	.135	.125	.472	.145	25.032	.170	.870	.202
30	.260	.127	.617	.148	.202	.173	27.072	.206
36	.387	.129	.765	.149	.375	.176	.278	.209
42	.516	.130	.914	.152	.551	.179	.487	.213
48	.546	.133	24.066	.154	.730	.182	.700	.217
54	.679	.134	.221	.157	.912	.185	.917	.221
60	.913		.378		26.097		28.138	

φ	78°		79°		80°		81°	
′	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.
00	28.138	.225	30.594	.273	33.603	.339	37.375	.430
06	.363	.229	.867	.279	.942	.346	.805	.442
12	.592	.234	31.146	.285	34.388	.355	38.247	.453
18	.826	.238	.431	.290	.643	.363	.700	.465
24	29.064	.243	.721	.297	35.006	.371	39.165	.477
30	.307	.247	32.018	.303	.377	.380	.642	.491
36	.554	.253	.321	.310	.757	.390	40.133	.504
42	.807	.257	.631	.317	36.147	.399	.637	.518
48	30.064	.262	.948	.324	.546	.410	41.155	.534
54	.326	.268	33.272	.331	.956	.419	.689	.549
60	.594		.603		37.375		42.238	

Table III. continued.

φ	82°		83°		84°		85°	
′	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.
	′		′		′		′	
00	42.238	.281	48.769	.387	58.065	.572	72.635	.960
03	.519	.285	49.156	.393	.637	.584	73.595	.991
06	.804	.289	.549	.400	59.221	.597	74.586	1.024
09	43.093	.293	.949	.408	.818	.612	75.610	1.057
12	.386	.298	50.357	.415	60.430	.625	76.667	1.093
15	.684	.302	.772	.423	61.055	.641	77.760	1.132
18	.986	.307	51.195	.430	.696	.656	78.892	1.174
21	44.293	.312	.625	.439	62.352	.672	80.064	1.216
24	.605	.317	52.064	.447	63.024	.689	81.280	1.262
27	.922	.322	.511	.456	.713	.706	82.542	1.311
30	45.244	.327	.967	.464	64.419	.725	83.853	1.364
33	.571	.332	53.431	.474	65.144	.743	85.217	1.422
36	.903	.338	.905	.484	.887	.764	86.639	1.482
39	46.241	.343	54.389	.493	66.651	.784	88.121	1.549
42	.584	.349	.882	.503	67.435	.806	89.670	1.620
45	.933	.355	55.385	.514	68.241	.829	91.290	1.698
48	47.288	.361	.899	.524	69.070	.853	92.988	1.784
51	.649	.367	56.423	.536	.922	.878	94.772	1.876
54	48.016	.373	.959	.547	70.800	.904	96.648	1.978
57	.389	.380	57.506	.559	71.704	.929	98.626	2.089
60	.769		58.065		72.635		100.715	

IV. *Table of the* decrements *in the Azimuths of the Points of Equilibrium corresponding to an* increment *of* 1° *in the Temperature of the Magnets, calculated at intervals of* 6′ *in the Azimuths from* 74° *to* 82°, *and of* 3′ *in those from* 82° *to* 86°; *the distances of the centres of the Magnets from the centre of the Needle being* 21.52 *inches: to be applied to the correction of the observed Azimuths when the observed Temperature of the Magnets is* below *the Mean Temperature to which the observations are to be reduced.*

φ	74°		75°		76°		77°	
′	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.
00	21.249	.110	22.421	.126	23.770	.146	25.338	.170
06	.359	.112	.547	.128	.916	.149	.508	.173
12	.471	.113	.675	.130	24.065	.151	.681	.176
18	.584	.115	.805	.132	.216	.153	.857	.179
24	.699	.116	.937	.134	.369	.155	26.036	.181
30	.815	.118	23.071	.136	.524	.158	.217	.184
36	.933	.119	.207	.137	.682	.160	.401	.188
42	22.052	.122	.344	.140	.842	.163	.589	.190
48	.174	.123	.484	.142	25.005	.165	.779	.193
54	.297	.124	.626	.144	.170	.168	.972	.197
60	.421		.770		.338		27.169	

φ	78°		79°		80°		81°	
′	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.	Δ φ	Dif.
00	27.169	.200	29.334	.238	31.923	.287	35.063	.351
06	.369	.203	.572	.243	32.210	.292	.414	.358
12	.572	.207	.815	.247	.502	.298	.772	.366
18	.779	.211	30.062	.251	.800	.304	36.138	.374
24	.990	.214	.313	.256	33.104	.310	.512	.382
30	28.204	.218	.569	.261	.414	.316	.894	.391
36	.422	.222	.830	.265	.730	.323	37.285	.399
42	.644	.226	31.095	.271	34.053	.330	.684	.408
48	.870	.230	.366	.276	.383	.337	38.092	.417
54	29.100	.234	.642	.281	.720	.343	.509	.427
60	.334		.923		35.063		.936	

Table IV. continued.

φ	82°		83°		84°		85°	
′	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.	Δφ	Dif.
	′		′		′		′	
00	38.936		43.805		50.075		58.371	
03	39.153	.217	44.081	.276	.435	.360	.855	.484
06	.373	.220	.361	.280	.801	.366	59.347	.492
09	.595	.222	.644	.283	51.171	.370	.847	.500
12	.820	.225	.931	.287	.547	.376	60.355	.508
15	40.047	.227	45.222	.291	.928	.381	.871	.516
18	.277	.230	.516	.294	52.315	.387	61.395	.524
21	.510	.233	.815	.299	.707	.392	.928	.533
24	.745	.235	46.117	.302	53.105	.398	62.469	.541
27	.983	.238	.423	.306	.508	.403	63.020	.551
30	41.224	.241	.733	.310	.918	.410	.579	.559
33	.468	.244	47.047	.314	54.334	.416	64.148	.569
36	.715	.247	.366	.319	.756	.422	.726	.578
39	.965	.250	.688	.322	55.184	.428	65.313	.587
42	42.218	.253	48.016	.327	.619	.435	.911	.598
45	.474	.256	.347	.332	56.060	.441	66.518	.607
48	.734	.260	.683	.336	.508	.448	67.136	.618
51	.997	.263	49.024	.341	.963	.455	.765	.629
54	43.263	.266	.369	.345	57.425	.462	68.404	.639
57	.532	.269	.720	.351	.894	.469	69.055	.651
60	.805	.273	50.075	.355	58.371	.477	69.717	.662

I now proceed to the observations for the correction of which these tables were calculated. My principal object in making these observations, was to ascertain how far they would enable me to determine the diurnal changes in the terrestrial magnetic intensity, and whether a series of such observations would not afford very accurate measures of such changes; and I have before stated that I made them both within-doors and in the open air, in order to ascertain whether I had, in my former paper, assigned the true cause of the apparent anomalies which were noticed by Mr. BARLOW and myself in these different situations.

The first observations were made in-doors, in the same

room as those from the 20th to the 27th of April, described in my former paper. The compass was placed on an horizontal table, with its centre at the distance of 5 feet from the middle of the only window in the room, and which was nearly in the direction of the magnetic meridian from it. I mention this circumstance, not that I myself consider it of importance, but as a datum for those who may be disposed to attribute the diurnal changes in the direction of the needle to the influence of light. The only iron in the room is a large lock to the door and the weights to the window, which, when the observations were made, were always in the same position. The magnets were placed on the table with their axes, as nearly as I could adjust them, in the meridian, the north pole of each being, as I have before mentioned, towards the north, and the distances of their centres from the centre of the needle 21.21 inches.

The method which I at first adopted for determining the changes that took place in the temperature of the magnets, was by placing a thermometer with the bulb *near* the southern extremity of the north magnet. In this manner I continued to observe for five days: I then placed the bulb of the thermometer *on* the southern extremity of the north magnet; and continued the observations for five days longer. I consider that the changes in the thermometer would, in either case, very nearly indicate the changes in the temperature of the magnets, especially as those changes were very gradual, and did not exceed 10° during the whole time in which the observations were made.

In the present state of our knowledge respecting the causes of magnetical phænomena, it is difficult to say how far

atmospheric changes may influence the direction and intensity of the terrestrial magnetism; I consider, therefore, in order that all possible information should be derived from a series of such observations as I am about to describe, that they should be accompanied with very precise observations of all the atmospheric changes which take place, particularly those of an electric nature. It was not always in my power to note these with the requisite precision; and as the observations were not continued for a sufficient length of time to enable me to derive any thing from those which I made on the force and direction of the wind, the character and appearance of the clouds, &c. I omit them: I have however inserted the changes which I noticed in the state of the barometer.

Table of observations, made within doors, *on the Diurnal Changes in the positions of the Points of Equilibrium at which a Magnetic Needle was retained by the joint action of Terrestrial Magnetism and of two bar Magnets, having their axes horizontal and in the magnetic meridian, and their centres at the distance* 21.21 *inches from the centre of the Needle.*

Date and Time of Observation.		Temp. of the Magnets.	Point. Equilibrium.			Barometer.	Therm. attached.
			Westerly	asterly.	South.		
	h. m.		° ′	° ′	° ′		°
22d May, 1823. Morning.	6 00	59.75	82 06	80 24	0 04 W	29.75	59.66
	7 30	60.00	82 44	81 06	0 02 W	29.75	59.66
	8 55	60.75	82 20	81 06	0 06 W	29.74	60.00
	10 30	60.75	82 34	81 30	0 14 W	29.75	60.50
Afternoon.	0 00	61.00	82 04	80 00	0 44 W	29.75	60.50
	1 45	61.15	81 38	79 16	0 34 W	29.76	59.75
	3 05	60.75	81 20	80 20	0 10 W	29.76	60.00
	4 35	60.50	81 34	80 40	0 06 E	29.75	60.25
	6 10	60.00	81 20	80 46	0 04 E	29.80	57.75
	7 40	59.50	81 50	80 52	0 02 W	29.82	58.00
	9 30	58.75	82 04	80 32	0 10 W	29.86	58.50

Table of observations made within doors, &c.

Date and time of Observation.		Temperature of the Magnets.	Points of Equilibrium. Westerly.	Points of Equilibrium. Easterly.	Points of Equilibrium. South.	Barometer.	Therm. attached.
	h. m.		° ′	° ′	° ′		° ′
23d May, 1823. Morning.	6 30	58.50	83 14	82 22	0 14 E	29.82	59.00
	7 35	59.00	83 38	82 26	0 08 E	29.82	58.75
	9 10	59.00	84 38	83 36	0 18 E	29.81	58.00
	10 35	60.00	84 40	83 20	0 00	29.82	58.50
Afternoon	0 10	59.80	83 12	81 22	0 28 W	29.85	59.00
	1 35	59.75	82 38	80 20	0 34 W	29.85	59.20
	3 05	59.50	82 50	80 20	0 28 W	29.87	59.33
	4 35	59.25	82 42	80 04	0 18 W	29.89	59.50
	6 05	59.50	83 12	80 42	0 06 W	29.91	60.25
	7 27	59.00	82 40	81 00	0 00	29.94	59.00
	9 40	58.00	83 26	81 18	0 12 W	29.98	58.00
24th May. Morning.	6 00	57.00	84 12	82 26	0 02 E	29.98	57.66
	7 30	57.66	85 04	83 34	0 04 W	29.97	55.66
	9 05	57.75	86 46	84 14	0 00	29.97	55.80
	10 30	58.75	86 00	84 14	0 16 W	29.96	56.50
Afternoon.	0 05	60.00	84 26	82 02	0 28 W	29.96	59.25
	1 35	59.90	82 46	81 04	0 38 W	29.96	61.10
	3 00	60.80	83 56	81 32	0 34 W	29.96	61.20
	4 30	60.20	81 08	79 42	0 26 W	29.96	60.00
	6 00	60.20	82 10	80 38	0 06 W	29.95	60.00
	7 35	59.30	81 48	80 38	0 14 W	29.93	59.50
	9 35	58.75	82 34	80 42	0 16 W	29.91	57.75
25th May. Morning.	6 00	58.25	83 10	82 08	0 04 E	29.77	59.10
	7 35	59.33	84 12	82 10	0 04 E	29.74	60.10
	9 05	60.10	84 24	82 36	0 12 W	29.74	61.00
	10 45	61.00	82 46	80 38	0 26 W	29.74	62.50
Afternoon.	0 00	61.00	82 26	80 26	0 34 W	29.74	62.50
	1 40	61.10	82 06	80 00	0 28 W	29.74	62.40
	3 00	61.50	81 32	79 34	0 26 W	29.74	61.75
	4 30	No observation.					
	6 00	61.00	81 20	79 26	0 24 W	29.74	61.50
	7 30	60.50	81 36	80 04	0 18 W	29.74	60.25
	9 45	59.75	82 04	80 42	0 14 W	29.74	61.25
26th May. Morning.	6 00	59 00	83 26	81 32	0 06 W	29.75	59.75
	7 30	59 75	83 10	81 52	0 08 E	29.75	59.50
	9 00	59 75	84 36	83 20	0 16 E	29.74	59.25
	10 30	60 50	86 00	83 50	0 14 W	29.75	59.75
Afternoon.	0 10	61 10	83 12	80 42	0 46 W	29.76	61.75
	1 30	61 50	82 40	80 22	0 54 W	29.77	62.50
	3 00	61 50	82 00	79 52	0 36 W	29.80	61.75
	5 00	61 50	81 42	80 08	0 14 W	29.81	62.10
	5 55	61 50	81 10	79 38	0 14 W	29.83	62.00
	7 40	61.00	81 00	79 38	0 10 W	29.85	61.50
	9 30	60.00	81 34	79 50	0 16 W	29.89	60.00

Table of observations made within dooors, &c.

Date and time of Observation.		Temperature of the Magnets.	Points of Equilibrium.			Barometer.	Therm. attached.
			Westerly.	Easterly.	South		
	h. m.	°	° ′	° ′	° ′	°	°
27th May, 1823. Morning.	6 00	59.00	83 00	81 52	0 04 E	29.98	59.25
	7 35	60.00	82 36	81 38	0 14 E	29.99	57.50
	9 05	60.90	83 26	82 18	0 06 E	30.00	59.80
	10 30	61.50	83 34	82 20	0 10 W	30.01	61.25
Afternoon.	0 00	62.50	82 36	80 58	0 40 W	30.02	63.80
	1 30	63.00	81 34	79 54	0 46 W	30.02	64.30
	3 00	62.60	81 32	79 44	0 34 W	30.02	64.40
	4 30	61.75	81 34	80 08	0 10 W	30.02	61.75
	6 00	61.00	81 40	80 38	0 04 W	30.02	60.25
	7 30	60.20	81 30	80 04	0 06 W	30.02	59.50
	9 40	60.00	81 58	80 20	0 08 W	30.03	59.50
	11 25	60.00	81 40	80 00	0 02 E	30.04	60.60
28th May. Morning.	6 07	59.75	82 30	81 42	0 14 E	30.07	61.00
	7 30	60.10	83 12	81 52	0 04 E	30.07	59.00
	9 00	60.50	83 28	81 52	0 00 W	30.09	62.00
	10 25	61.00	83 40	81 42	0 24 W	30.09	64.75
Afternoon.	0 00	61.20	83 34	81 22	0 44 W	30.09	66.00
	1 30	61.50	82 04	80 06	0 52 W	30.09	66.25
	3 00	62.30	80 48	79 04	0 46 W	30.09	66.90
	4 30	63.25	80 56	79 46	0 12 W	30.09	66.50
	6 00	63.00	80 44	79 38	0 08 W	30.09	65.50
	7 20	62.75	80 10	79 20	0 16 W	30.09	64.00
	9 30	61.25	81 34	79 58	0 12 W	30.10	58.80
	11 20	61.00	81 54	80 14	0 16 W	30.15	61.80
29th May. Morning.	6 20	60.00	82 46	81 18	0 06 W	30.18	60.60
	7 30	60.40	82 52	81 18	0 04 W	30.19	60.30
	9 00	61.00	82 58	81 24	0 04 W	30.19	55.00
	10 30	62.75	83 24	82 10	0 20 W	30.19	57.00
Afternoon.	0 10	63.00	82 46	80 48	0 22 W	30.19	59.25
	1 30	62.00	82 42	80 16	0 36 W	30.19	60.75
	3 00	62.30	81 52	79 38	0 32 W	30.19	62.20
	4 30	63.75	80 46	79 16	0 18 W	30.19	63.25
	6 10	63.30	80 38	78 56	0 20 W	30.19	63.00
	7 30	62.00	80 54	79 18	0 18 W	30.19	60.25
	9 45	60.75	81 24	79 36	0 10 W	30.19	58.50
	11 20	60.50	81 42	80 12	0 10 W	30.20	60.50
30th May. Morning.	6 05	60.30	82 10	80 50	0 02 E	30.22	60.50
	7 30	61.30	82 12	80 38	0 02 W	30.23	59.40
	9 00	62.00	82 30	81 40	0 12 E	30.23	62.75
	10 30	62.25	82 46	81 50	0 02 W	30.24	64.25
Afternoon.	0 10	64.00	82 04	80 22	0 40 W	30.24	65.50
	1 45	63.50	81 58	79 42	0 36 W	30.24	65.40
	3 00	63.75	80 52	78 44	0 28 W	30.24	65.75
	4 30	64.00	80 10	78 38	0 06 W	30.24	66.75
	5 55	63.50	79 50	78 16	0 18 W	30.24	66.00
	7 22	63.20	80 28	78 44	0 16 W	30.24	65.00
	9 40	62.25	80 52	79 04	0 16 W	30.25	63.00
	11 10	62.00	81 00	79 20	0 16 W	30.26	63.25

Table of observations made within doors, &c.

Date and time of observation.		Temperature of the Magnets.	Points of Equilibrium.			Barometer.	Therm. attached.
			Westerly.	Easterly.	South.		
	h. m.	°	° ′	° ′	° ′	° ′	°
31st May, 1823. Morning.	6 10	62.00	81 50	80 12	0 04 E	30.27	63.00
	7 30	62.50	81 56	80 26	0 08 E	30.28	62.00
	9 00	63.00	82 56	81 18	0 06 E	30.28	64.50
	10 30	63.40	82 32	80 40	0 16 W	30.27	65.75
Afternoon.	0 00	63.66	81 32	79 34	0 24 W	30.28	66.00
	1 30	63.75	81 04	79 02	0 32 W	30.28	66.33
	3 00	64.20	80 10	78 38	0 30 W	30.28	66.33
	4 30	65.00	80 10	78 36	0 20 W	30.28	67.75
	5 55	65.00	79 40	78 24	0 16 W	30.27	65.66
	7 30	64.66	79 44	78 20	0 16 W	30.27	63.50
	9 25	63.33	79 54	78 50	0 16 W	30.26	61.66
	11 30	63.00	80 28	79 00	0 16 W	30.26	64.00

In all the observations which I have made, I have considered the magnetic meridian to be the line of direction of a needle at the time when that direction is most stationary, that is at about seven o'clock in the evening; and in arranging the magnets for the foregoing and similar observations, I have not only always found much difficulty, but have seldom succeeded, in determining so accurately the axes of the magnets, and adjusting them so precisely in the meridian, that, at that time, the needle should be held in equilibrio exactly at south, and also at points towards the west and east equidistant from the north, which evidently ought to be the case with a perfect adjustment. Partly from this difficulty in adjusting the magnets, of which those who have attempted similar arrangements will be best aware, and partly from the changes which, even during the evening, take place in the direction and intensity of the terrestrial forces, the east and west points of equilibrium, in the foregoing observations, are not, during the evening, at equal

distances from the north, nor is the south point exactly at south. In order to reduce the situations of these points to their distances from what ought to be considered as their meridian, I take the mean of the azimuths of the westerly point at the evening observations, which is 81° 27′, and also of the corresponding azimuths of the easterly points, 79° 57′; half their difference will be the mean error in the point which has been considered as zero of the compass with reference to these points: so that if 45′ be subtracted from each of the azimuths of the westerly point, and added to those of the easterly, these points will be reduced very nearly to what would have been their positions had all the adjustments been perfect. With regard to the southerly point of equilibrium, the mean of the evening observations gives its position 12′ W; this therefore should be subtracted from the westerly and added to the easterly, in order to reduce the observed deviations to those from the meridian. These reductions I have made in the following table, preparatory to the reduction to be made in consequence of the changes in the temperature of the magnets.

Table of the preceding observations reduced to their Mean Magnetic Meridian.

| May 22. | | |

To reduce these observed positions of the points of equilibrium to their true positions, that is, those which they would have had if the temperature of the magnets had been the same at each of the observations, it is necessary to apply a correction by means of Tables I. and II.; and that the nature of this reduction may be evident, I shall give an instance of the process at length of applying the tables to the correction of the observations, when the temperature at which they were made was *below* the standard temperature, and also when it was *above* that temperature. As the observations were made with the magnets at temperatures varying nearly equally *above* and *below* 60°, I consider that, the standard temperature to which to reduce them. The two following are instances of this reduction.

1st. Observed temperature *below* the standard temperature.

24th May, 6h 00m A.M.	Westerly.	Easterly.	
Points of Equilibrium	83°27	83 11	at temp. 57°
Correction for 1° Temp. Table II.	— 47.002	— 45.397	
Points of Equilibrium	82 39.998	82 25.603	at temp. 58°
Correction for 1° Temp.	— 42.581	— 41.380	
Points of Equilibrium	81 57.417	81 44.223	at temp. 59°
Correction for 1° Temp.	— 39.244	— 38.316	
Reduced Poiuts of Equilibrium	81 18.173	81 05.907	at standard temp. 60°.

2nd. Observed temperature *above* the standard temperature.

29th May, Noon.	Westerly.	Easterly.	
Points of Equilibrium	82° 00′	81° 34′	at temp. 63°
Correction for 1° Temp. Table I.	+ 42.820	+ 40.517	
Points of Equilibrium	82 42.820	82 14.517	at temp. 62°
Correction for 1° Temp.	+ 47.329	+ 44.241	
Points of Equilibrium	83 30.149	82 58.758	at temp. 61°
Correction for 1° Temp.	+ 53.713	+ 49.296	
Reduced Points of Equilibrium	84 23.862	83 48.054	at standard temp. 60°.

By processes similar to these, making use of Table I. or II. according as the observed temperature of the magnets is above or below the standard temperature 60°, the observed positions of the points of equilibrium are reduced to what would have been their positions had the temperature of the magnets been 60° at each observation.

A. ***Table of the positions of the Points of Equilibrium at which a Magnetic Needle was retained at different hours during the day, by the joint action of two bar Magnets and of Terrestrial Magnetism, reduced to their true positions at the Standard Temperature*** (60°) ***of the Magnets.*** Note. The observations were made within doors.

Time of Observation.	May 22. Difference of Temperature corrected for	May 22. Points of Equilibrium. West.	May 22. Points of Equilibrium. East.	23. Difference of Temperature corrected for	23. Points of Equilibrium. West.	23. Points of Equilibrium. East.	24. Difference of Temperature corrected for	24. Points of Equilibrium. West.	24. Points of Equilibrium. East.	25. Difference of Temperature corrected for	25. Points of Equilibrium. West.	25. Points of Equilibrium. East.	26. Difference of Temperature corrected for	26. Points of Equilibrium. West.	26. Points of Equilibrium. East.	Mean true positions of the Points of Equilibrium. Westerly.	Easterly.	South, as before.
h. m.	°	° ′	° ′	°	° ′	° ′	°	° ′	° ′	°	° ′	° ′	°	° ′	° ′	° ′	° ′	° ′
6 00	+0.25	81 12	81 00	+1.50	81 28	82 01	+3.00	81 18	81 06	+1.75	81 15	81 39	+1.00	81 58	81 36	81 26.2	81 28.4	0 14.0 E
7 30	0.00	81 59	81 51	+1.00	82 09	82 27	+2.33	82 25	82 25	+0.67	82 56	82 26	+0.25	82 15	82 26	82 20.8	82 19.8	0 14.8 E
9 00	−0.75	82 05	82 22	+1.00	83 03	83 28	+2.25	83 41	82 58	−0.10	83 45	83 26	+0.25	83 39	83 52	83 14.6	83 13.2	0 15.2 E
10 30	−0.75	82 20	82 48	0.00	83 55	84 05	+1.25	84 01	83 47	−1.00	82 44	82 03	−0.50	85 54	85 08	83 46.8	83 34.2	0 02.0 W
Noon.	−1.00	81 58	81 22	+0.20	82 19	81 59	0.00	83 41	82 47	−1.00	82 22	81 50	−1.10	83 18	82 12	82 43.6	82 02.0	0 24.0 W
1 30	−1.15	81 36	80 40	+0.25	81 43	80 56	+0.10	81 57	81 45	−1.10	82 05	81 26	−1.50	83 01	82 06	82 04.4	81 22.6	0 25.6 W
3 00	−0.75	81 02	81 34	+0.50	81 45	80 47	−0.80*	83 52	82 53	−1.50	81 44	81 13	−1.50	82 15	81 33	81 41.5	81 16.8	0 14.8 W
4 30	−0.50	81 07	81 45	+0.75	81 28	80 23	−0.20	80 30	80 34	No observation.			−1.50	81 55	81 50	81 15.0	81 08.0	0 01.0 W
6 00	0.00	80 35	81 31	+0.50	82 06	81 08	−0.20	81 33	81 31	−1.00	81 11	80 46	−1.50	81 19	81 17	81 20.8	81 14.4	0 02.8 E
7 30	+0.50	80 47	81 18	+1.00	81 16	81 07	+0.70	80 38	80 57	−0.50	81 01	80 58	−1.00	80 50	80 58	80 56.2	81 05.4	0 03.2 E
9 30	+1.25	80 33	80 32	+2.00	81 19	80 46	+1.25	81 03	80 41	+0.25	81 10	81 18	0.00	80 49	80 35	80 58.8	80 46.2	0 01.6 W

Time of Observation.	May 27. Difference of Temperature corrected for	May 27. Points of Equilibrium. West.	May 27. Points of Equilibrium. East.	28. Difference of Temperature corrected for	28. Points of Equilibrium. West.	28. Points of Equilibrium. East.	29. Difference of Temperature corrected for	29. Points of Equilibrium. West.	29. Points of Equilibrium. East.	30. Difference of Temperature corrected for	30. Points of Equilibrium. West.	30. Points of Equilibrium. East.	31. Difference of Temperature corrected for	31. Points of Equilibrium. West.	31. Points of Equilibrium. East.	Mean true positions of the Points of Equilibrium. Westerly.	Easterly.	South, as before.
6 00	+1.00	81 35	81 55	+0.25	81 35	82 17	0.00	82 01	82 03	−0.30	81 37	81 47	−2.00	82 24	82 15	81 50.4	82 03.4	0 15.6 E
7 30	0.00	81 51	82 23	−0.10	82 31	82 41	−0.40	82 24	82 20	−1.30	82 20	82 16	−2.50	82 54	82 54	82 24.0	82 30.8	0 16.0 E
9 00	−0.90	83 23	83 48	−0.50	83 07	83 00	−1.00	82 57	82 53	−2.00	83 12	84 05	−3.00	84 42	84 29	83 28.2	83 39.0	0 16.0 E
10 30	+1.50	84 04	84 24	−1.00	83 44	83 13	−2.75	85 06	85 31	−2.25	83 44	84 29	−3.40	84 29	83 58	84 13.4	84 19.0	0 02.4 W
Noon.	+2.50	83 45	83 35	−1.20	83 48	83 00	−3.00	84 25	83 47	−4.00	84 21	84 04	−3.67	83 17	82 41	83 55.2	83 25.4	0 22.0 W
1 30	−3.00	82 49	82 37	−1.50	82 19	81 48	−2.00	83 26	82 20	−3.50	83 45	82 44	−3.75	82 44	82 03	83 00.6	82 18.4	0 28.4 W
3 00	−2.60	82 29	82 07	−2.30	81 25	81 09	−2.30	82 40	81 48	−3.75	82 29	81 41	−4.20	81 53	81 51	82 11.2	81 43.2	0 22.0 W
4 30	−1.75	81 56	82 01	−3.25	82 13	82 38	−3.75	82 21	82 21	−4.00	81 45	81 43	−5.00	82 27	82 23	82 08.4	82 13.2	0 02.4 W
6 00	−1.00	81 32	82 02	−3.00	81 47	82 17	−3.30	81 52	81 38	−3.50	81 02	80 57	−5.00	81 46	82 04	81 35.8	81 47.6	0 01.4 W
7 30	−0.20	80 52	80 56	−2.75	80 58	81 45	−2.00	81 20	81 13	−3.20	81 36	81 20	−4.67	81 38	81 47	81 16.8	81 24.2	0 02.4 W
9 30	0.00	81 13	81 05	−1.25	81 36	81 30	−0.75	81 06	80 47	−2.25	81 28	81 07	−3.33	81 01	81 32	81 16.8	81 12.2	0 00.4 W
11 20	0.00	80 55	80 45	−1.00	81 43	81 33	−0.50	81 16	81 16	−2.00	81 27	81 23	−3.00	81 28	81 31	81 21.8	81 17.6	0 00.8 E

* In taking the mean, I reject this observation as evidently irregular.

To obtain from these corrected observations the diurnal variation of the terrestrial magnetic intensity, I take half the sum of the mean easterly and westerly arcs at different hours during the day as the mean azimuths of the points of equilibrium at those hours, and substituting these azimuths successively for φ in the equation (α),

$$M - F(.004690814 + 000829329 \cos.^2 \varphi) = 0,$$

I obtain the values of M in terms of F at those hours: dividing each of these values by the minimum value of M, which in every case appears to happen at about $10^h\ 30^m$ in the morning, I obtain the relative terrestrial magnetic intensities at the times of observation. These results are contained in the following table.

B. *Table of the mean Terrestrial Magnetic Intensities at different hours during the day, deduced from the preceding observations.*
Note. The observations were made within doors.

Time of Observation.	Mean of the Observations of May 25, 26, 24, 25, 26.		Mean of the Observations of May 27, 28, 29, 30, 31.		Mean of the two sets.
	Azimuth of the Points of Equilibrium.	Terrestrial Magnetic Intensity.	Azimuth of the Points of Equilibrium.	Terrestrial Magnetic Intensity.	Terrestrial Magneric Intensity.
h. m.	° ′		° ′		
6 00	81 27.3	1.00175	81 56.9	1.00170	1.00173
7 30	82 19.9	1.00100	82 27.4	1.00128	1.00114
9 00	83 13.9	1.00031	83 33.6	1.00046	1.00039
10 30	83 40.5	1.00000	84 16.2	1.00000	1.00000
Noon.	82 22.8	1.00096	83 40.3	1.00038	1.00067
1 30	81 43.5	1.00151	82 39.5	1.00112	1.00132
3 00	81 29.1	1.00173	81 57.2	1.00170	1.00172
4 30	81 11.5	1.00199	82 10.8	1.00151	1.00175
6 00	81 17.7	1.00190	81 41.7	1.00192	1.00191
7 30	81 00.9	1.00216	81 20.5	1.00224	1.00220
9 30	80 52.6	1.00229	81 14.5	1.00233	1.00231
11 20			81 19.7	1.00225	1.00225

From the mean obtained here, it appears that the terrestrial magnetic intensity was the least between 10 and 11 o'clock in the morning, the time, nearly, when the sun was on the magnetic meridian; that it increased from this time until between 9 and 10'clock in the evening; after which it decreased, and continued decreasing during the morning until the time of the minimum.

Having by this reduction of the observations made within doors, determined the nature of the changes in the direction of the needle in that situation, independent of the changes which took place in the temperature of the magnets, and thence deduced the diurnal changes in the intensity of the terrestrial forces acting upon the needle, I shall now detail similar observations which I made in the open air, for the purpose of comparing with them, when these had also been cleared of the effects due to changes in the temperature of the magnets, in order to determine how far there was any thing anomalous in the directions of the needle when in doors and when in the open air. I have already mentioned that, for the purpose of making these observations, the apparatus was placed on a table fixed firmly in my garden, the magnets being placed in earthen pans containing water. The observations were made in the same manner as those in doors, excepting that, as the magnets were here liable to greater changes of temperature, their temperatures were noticed at the beginning, and also at at the conclusion of each of the observations: they are contained in the following table, where the time set down is that at which the observation commenced, the time occupied in making the whole of each being from four to six minutes.

Table of observations, made in the open air, *on the Diurnal Changes in the positions of the Points of Equilibrium at which a Magnetic Needle was retained by the joint action of Terrestrial Magnetism, and of two bar Magnets, having their axes horizontal and in the Magnetic Meridian, and their centres at the distance* 21.52 *inches from the centre of the needle.*

Date and Time of Observation.		Temperature of the Magnets.		Points of Equilibrium.			Temperature of the Magnets.		Mean Temperature of the Magnets.	Barom.	Therm. attached.
		North.	South.	West.	East.	South.	North.	South.			
	h. m.	°	°	° ′	° ′	°	°	°	°		°
19th June, 1823. Afternoon. Morning.	6 05	55.2	54.0	85 50	85 02	0 20 E	54.8	54.0	54.50	30.23	58.2
	7 24	55.2	55.2	85 18	84 24	0 20 E	55.6	55.6	55.40	30.23	57.1
	8 53	57.3	56.7	85 32	83 38	0 06 E	57.4	57.0	57.10		
	10 30	No observation.									
	0 05	61.8	60.0	82 06	80 18	0 34 W	61.8	60.0	60.90	30.20	58.2
	1 26	62.5	61.0	81 02	78 46	0 44 W	62.5	61.0	61.75	30.19	59.8
	2 56	63.8	62.0	79 14	77 58	0 28 W	63.8	62.0	62.90	30.19	60.6
	4 26	63.0	61.0	80 00	78 58	0 20 W	62.8	61.0	61.95	30.18	59.5
	6 10	59.0	58.0	81 36	80 20	0 00	58.8	57.8	58.40	30.18	56.8
	7 25	56.0	55.0	83 08	82 06	0 06 E	55.8	55.0	55.45		
	9 00	55 6	55.8	82 34	81 30	0 12 W	55.3	55.5	55.55	30.17	55.7
20th June. Afternoon. Morning.	6 12	55.7	55.5	82 50	82 04	0 12 E	55.5	55.3	55.50		
	7 25	55.2	54.8	84 12	83 02	0 18 E	55.5	55.1	55.15	30.11	56.75
	9 00	66.25	64.0	77 52	78 08	0 18 E	66.25	64.0	65.13	30.11	60.25
	10 27	68.0	66.0	77 18	76 40	0 20 W	68.0	66.0	67.00	30.11	63.0
	0 12	71.0	68.8	77 04	74 42	0 26 W	71.0	68.8	69.90	30.09	64.3
	1 26	70.7	69.0	75 24	74 20	0 38 W	70.7	69.0	69.85	30.09	64.6
	3 00	70.0	68.4	75 20	74 02	0 32 W	69.8	68.3	69.14	30.08	65.1
	4 30	69.0	67.5	75 16	74 28	0 16 W	69.0	67.5	68.25	30.07	65.8
	6 00	67.6	66.3	75 42	74 54	0 02 W	67.6	66.3	66.95	30.07	64.8
	7 30	65.8	64.2	76 18	75 38	0 02 W	65.8	64.2	65.00	30.08	64.0
	9 00	61.5	60.1	79 14	77 38	0 10 W	61.4	60.0	60.73	30.09	60.0
21st June. Afternoon. Morning.	5 50	56.8	55.0	81 46	81 28	0 26 E	56.6	54.8	55.80	30.10	60.4
	7 26	56.3	55.7	81 46	81 20	0 22 E	56.1	55.7	55.95	30.13	57.5
	9 00	55.4	54.8	82 48	82 54	0 08 E	55.4	54.8	55.10	30.15	57.8
	10 26	56.5	56.0	82 48	82 36	0 06 W	56.7	56.0	56.30	30.15	57.7
	0 00	58.0	58.0	81 50	80 40	0 24 W	58.8	58.0	58.35	30.16	58.5
	1 30	60.0	58.8	80 08	78 52	0 26 W	60.0	58.8	59.40	30.16	58.6
	3 00	60.5	58.7	79 54	78 40	0 24 W	60.5	58.7	59.60	30.17	58.5
	4 30	59.3	57.6	79 10	78 32	0 04 W	59.3	57.6	58.45	30.17	57.4
	6 00	57.5	56.0	79 38	79 04	0 06 E	57.4	55.9	56.70	30.17	57.0
	7 30	55.5	54.5	81 00	80 32	0 04 E	55.5	54.5	55.00	30.18	55.7
	9 00	53.9	53.1	82 18	81 38	0 00	53.8	53.0	53.65	30.20	55.4

Observations made in the open air, &c.

Date and Time of Observation.		Temperature of the Magnets.		Points of Equilibrium.			Temperature of the Magnets.		Mean Temperature of the Magnets.	Barom.	Therm. attached.
		North.	South.	West.	East.	South.	North.	South.			
	h. m.	°	°	° ′	° ′	° ′	°	°	°		°
22d June, 1823. Morning.	6 00	No observation.									
	7 44	55.1	54.5	83 44	83 02	0 14 E	55.0	54.4	54.75		
	9 07	65.5	64.0	78 36	78 04	0 08 E	65.3	64.0	64.70	30.21	55.0
	10 30	60.0	58.4	82 34	81 18	0 06W	59.9	58.3	59.15	30.21	54.5
	11 58	57.5	55.6	83 52	82 24	0 26W	57.4	55.5	56.50	30.22	55.8
Afternoon.	1 29	56.3	54.8	83 18	81 54	0 48W	56.5	54.8	55.60	30.22	56.7
	2 58	57.0	55.8	81 32	80 34	0 40W	57.0	55.8	56.40	30.23	57.4
	4 21	57.0	56.0	81 52	80 44	0 18W	57.0	56.0	56.50	30.20	56.7
	5 57	54.8	54.2	82 38	8 48	0 00	54.8	54.2	54.50	30.20	56.2
	7 28	53.5	52.4	83 44	8 08	0 06W	53.5	52.2	52.90	30.19	56.1
	8 53	52.0	51.0	84 54	8 52	0 00	52.0	51.0	51.50	30.19	56.4
23d June. Morning.	6 01	54.7	53.7	82 52	82 46	0 10 E	54.5	53.4	54.08	30.16	55.3
	7 28	55.0	54.5	83 52	82 58	0 20 E	55.2	54.5	54.80	30.15	54.8
	8 55	56.25	55.4	82 52	82 46	0 02W	56.25	55.4	55.83	30.14	55.5
	10 25	56.5	54.75	83 10	82 50	0 14W	56.5	54.75	55.63	30.14	55.0
	0 14	57.9	55.9	83 28	81 00	0 40W	58.2	56.2	57.05	30.10	56.2
Afternoon.	1 29	58.5	56.0	81 34	79 54	0 40W	58.5	56.0	57.25	30.11	55.4
	2 59	57.7	56.5	80 52	79 20	0 28W	57.7	56.5	57.10	30.11	56.1
	4 30	No observation.									
	6 00	No observation.									
	7 42	52.3	51.5	83 08	82 34	0 04W	52.3	51.5	51.90	30.10	56.7
	9 00	50.8	50.6	84 22	83 44	0 00	50.8	50.4	50.65	30.10	52.8

The mean of the azimuths of the westerly point at $7^h\ 30^m$ in the evening is 81° 28′, and of the easterly at the same time 80° 48′; so that to reduce the situations of the westerly and easterly points to their distances from what ought to be considered as their meridian, 20′ must be subtracted from each of the azimuths of the westerly point, and added to each of those of the easterly, similarly to what was done with the observations made in doors. The mean of the observations gives the position of the south point at the same hour 0′.4 W., or so nearly in the meridian, that the observations of this point require no reduction. The observed

azimuths, so reduced to their mean meridian, are to be corrected for the difference between the standard temperature and that of the magnets. By means of tables III. and IV, repeating the processes described for the reduction of the observations made in doors to the standard temperature 60°, I reduce these observed positions of the points of equilibrium to what would have been their positions had the temperature of the magnets been 60° at each of the observations. These reductions are successively effected in the two following tables.

Table of the preceding Observations reduced to their Mean Magnetic Meridian.

June 19					20				21.				22.				23.				Mean Positions of the South Point.
Time of Observation.	Mean Temperature of the Magnets.	Points of Equilibrium.			Mean Temperature of the Magnets.	Points of Equilibrium.			Mean Temperature of the Magnets.	Points of Equilibrium.			Mean temperature of the Magnets.	Points of Equilibrium.			Mean Temperature of the Magnets.	Points of Equilibrium.			
		West.	East.	South.		West.	East.	South.		West.	East.	South.		West.	East.	South.		West.	East.	South.	
h. m.	°	° ′	° ′	° ′	°	° ′	° ′	°	°	°	°	°	°	° ′	° ′	°	°	°	°	°	°
6 00	54.50	85 30	85 22	0 20 E	55.50	82 30	82 24	0 12 E	55.80	81 26	81 48	0 26 E		No observation.			54.08	82 32	83 06	0 10 E	0 19.0 E
7 30	55.49	84 58	84 44	0 20 E	55.15	83 52	83 22	0 18 E	55.95	81 26	81 40	0 22 E	54.75	83 24	83 22	0 14 E	54.80	83 22	83 18	0 20 E	0 18.0 E
9 00	57.10	85 12	83 58	0 06 E	65.13	77 32	78 28	0 18 E	55.10	82 28	83 14	0 08 E	64.70	78 16	78 24	0 08 E	55.83	82 32	83 06	0 02W	0 11.3 E
10 30		No observation.			67.00	76 58	77 00	0 20W	56.30	82 28	82 56	0 06W	59.15	82 14	81 38	0 06W	55.63	82 50	83 10	0 14W	0 10.7W
Noon.	60.90	81 46	80 38	0 34W	69.90	76 44	75 02	0 26W	58.35	81 30	81 00	0 24W	56.50	83 32	82 44	0 26W	57.05	83 08	81 20	0 40W	0 25.3W
1 30	61.75	80 42	79 06	0 44W	69.85	75 04	74 40	0 38W	59.40	79 48	79 12	0 26W	55.60	82 58	82 14	0 48W	57.25	81 14	80 14	0 40W	0 37.3W
3 00	62.90	78 54	78 18	0 28W	69.14	75 00	74 22	0 32W	59.60	79 34	79 00	0 24W	56.40	81 12	80 54	0 40W	57.10	80 32	79 40	0 28W	0 32.0W
4 30	61.95	79 40	79 18	0 20W	68.25	74 56	74 48	0 16W	58.45	78 50	78 52	0 04W	56.50	81 32	81 04	0 18W		No observation.			0 12.7W
6 00	58.40	81 16	80 40	0 00	66.95	75 22	75 14	0 02W	56.70	79 18	79 24	0 06 E	54.50	82 18	82 08	0 00		No observation.			0 01.3W
7 30	55.45	82 48	82 26	0 06 E	65.00	75 58	75 58	0 02W	55.00	80 40	80 52	0 04 E	52.90	83 24	83 28	0 06W	51.90	82 48	82 54	0 04W	0 01.3W
9 00	55.80	82 14	81 50	0 12W	60.75	78 54	77 58	0 10W	53.65	81 58	81 58	0 00	51.50	84 34	84 12	0 00	50.65	84 02	84 04	0 00	0 03.3W

C. *Table of the positions of the Points of Equilibrium at which a Magnetic Needle was retained at different hours during the day, by the joint action of two bar Magnets and of Terrestrial Magnetism, reduced to their true positions at the Standard Temperature* (60°) *of the Magnets.* Note. The observations were made in the open air.

June 19.				20.			21.			22.			23.			Mean true Positions of the Points of Equilibrium.		
Time of Observation.	Difference of Temperature corrected for	Points of Equilibrium.		Difference of Temperature corrected for	Points of Equilibrium.		Difference of Temperature corrected for	Points of Equilibrium.		Difference of Temperature corrected for	Points of Equilibrium.		Difference of Temperature corrected for	Points of Equilibrium.		Westerly.	Easterly.	South as before.
		West.	East.		West.	East.		West.	East.		West.	East.		West.	East.			
h. m.	°	° ′	° ′	°	° ′	° ′	°	° ′	° ′	°	° ′	° ′	°	° ′	° ′	° ′	° ′	° ′
6 00	+5.50	81 06	81 02	+4.50	79 48	79 44	+4.20	79 05	79 23		No observation.		+5.92	79 03	79 20	79 26.5	79 33.5	0 19.0 E
7 30	+4.60	81 22	81 13	+4.85	80 31	80 11	+4.05	79 10	79 21	+5.25	79.59	79 58	+5.20	80 06	79 56	79 53.3	79 50.0	0 18.0 E
9 00	+2.90	82 39	81 47	−5.13	80 03	81 14	+4.90	79 31	80 04	−4.70	80 43	80 53	+4.17	79 57	80 22	80 05.7	80 43.7	0 11.3 E
10 30	No observation.			−7.00	80 22	80 24	+3.70	80 10	80 31	+0.85	81 40	81 06	+4.37	80 04	80 19	80 44.0	80 40.3	0 10.7 W
Noon.	−0.90	82 23	81 10	−9.90	81 48	79 20	+1.65	80 31	80 03	+3.50	81 05	80 29	+2.95	81 07	79 38	81 08.0	79 57.3	0 25.3 W
1 30	−1.75	81 47	80 01	−9.85	79 21	78 49	+0.60	79 29	78 54	+4.40	80 09	79 36	+2.75	79 40	78 40	79 39.7	79 06.3	0 37.3 W
3 00	−2.90	80 26	79 45	−9.14	78 55	78 05	+0.40	79 22	78 48	+3.60	79 12	78 57	+2.80	79 02	78 16	79 09.7	78 36.7	0 32.0 W
4 30	−1.95	80 45	80 21	−8.25	78 23	78 13	+1.55	78 06	78 08	+3.50	79 31	79 08		No observation.		78 40.0	78 29.7	0 12.7 W
6 00	+1.60	80 20	79 47	−6.95	78 19	78 09	+3.30	77 43	77 48	+5.50	79 05	78 58		No observation.		78 22.3	78 18.3	0 01.3 W
7 30	+4.55	79 57	79 41	−5.00	78 07	78 07	+5.00	78 05	78 15	+7.10	79 01	79 04	+8.10	78 10	78 14	78 24.3	78 28.7	0 01.3 W
9 00	+4.20	79 43	79 24	−0.75	79 17	78 19	+6.35	78 26	78 26	+8.50	78 59	78 47	+9.35	78 18	78 19	78 54.0	78 30.7	0 03.3 W

The character of the diurnal changes in the positions of the points of equilibrium is very nearly the same for each day, but, in taking the mean, I can only make use of the observations of the 20th, 21st, 22d, since on the 19th no observation could be made at $10^h\ 30^m$, and the azimuths are all greater on this day than on any of the subsequent, and two observations were unavoidably omitted on the 23d.

Comparing the results with those obtained from the observations made in doors, we find them agree as nearly as could possibly be expected. From table A it appears, that when the observations were made in doors, the westerly point receded from the north until half past 10 o'clock in the morning, and approached the north during the remainder of the day until about 9 in the evening; and from table C, that when they were made in the open air, the westerly point receded from the north until about half past eleven in the morning, and approached it until six or seven in the evening, after which it again gradually receded. This is not a greater variation in the times of the maxima than we find on different days, either in the in-door observations, or in those in the open air. The easterly point appears to have receded from the north until about 10 o'clock in the morning, when the observations were made in doors and likewise when they were made in the open air; and to have approached it until between nine and ten in the evening in the former case, and until six in the latter.

Taking, as before, half the sum of the mean easterly and westerly arcs at different hours during the day as the mean azimuths of the points of equilibrium at those hours, and substituting these for ϕ in the equation (α_4),

$$M - F\,(.00448032 + .0007664093\ \cos.^4\phi) = 0,$$

I obtain the values of M in terms of F, at those hours; and dividing each of these values by the minimum value of M, I find, as before, the relative terrestrial magnetic intensities at the times of observation.

D. *Table of the mean Terrestrial Magnetic Intensities at different hours during the day, deduced from the preceding observations.* Note. The observations were made in the open air.

Time of Observation.	Mean of the Observations of June 20, 21, 22.	
	Azimuth of the Point of Equilibrium.	Terrestrial Magnetic Intensity.
h. m.		
6 00	79 30.0	1.00112
7 30	79 51.7	1.00061
9 00	80 24.7	1.00028
10 30	80 42.2	1.00000
Noon.	80 32.7	1.00015
1 30	79 28.0	1.00134
3 00	78 53.2	1.00188
4 30	78 34.8	1.00223
6 00	78 20.3	1.00251
7 30	78 26.5	1.00239
9 00	78 42.3	1.00209

From these it appears, that the minimum intensity happened nearly at the time the sun passed the magnetic meridian, and rather later than in May, which was also the case with the time of the sun's passage over the meridian:* the

* The diurnal variation, both in the direction of the needle and in the magnetic intensity, appears to have a reference to the position of the sun with regard to the magnetic meridian; it is therefore probable, that the sun is the principal cause of both

intensity increased until about six o'clock in the afternoon, after which time it appears to have decreased during the

these phœnomena. The circumstance of the situation of the magnetic pole in what appears to be, independent of elevation, the coldest region of the globe, supported as it is by the fact of a diminution of temperature causing an increase of magnetic intensity, would lead us to infer, that the effect produced by the sun is principally to be attributed to the heat developed by it; but should any periodical effects, corresponding to the time of the sun's rotation about its axis, be observable in the diurnal variation, we must suppose that the sun, like the earth, is endued with magnetism, and look for a cause of this magnetism, common to all the planets. Being engaged more than two years ago in making some experiments on the effects produced on the needle by unpolarized iron, I discovered that a peculiar polarity was imparted to the iron by simply making it revolve about an axis; and this naturally suggested the question to me, whether the magnetism of the earth, and consequently, that of the other planets and the sun, might not be owing to their rotation? From the effects which I have observed to be produced on iron by its rotation, it appears probable, if the magnetism of these bodies be not caused by their rotation, that at least the effects will be modified by, and, to a certain extent, dependent on such rotation. Since first observing the fact, that simple rotation will cause a peculiar polarity, if I may be allowed the expression, in iron, I have made a great variety of experiments on the subject, which have enabled me to trace the laws according to which this polarity in the iron affects a magnetic needle, independently of the effect produced by the mass. It would lead me to too great a length here to state the several effects that are produced by the rotation of iron, or the laws which govern them; but I will briefly mention one. Let us imagine a plane to pass through the centre of an horizontal needle, at right angles to the meridian, and making an angle with the horizon equal to the dip; then, if the plane of a circular plate of iron coincide with this plane, and the plate be fixed on an axis passing through its centre at right angles to its plane, so that it can be made to revolve in its own plane, the direction of the needle will be different, according as the several points of the plate are brought into any particular position by making it revolve in one direction or the opposite, excepting in four positions of the centre of the plate. If the centre of the plate be successively placed to the east or west of the centre of the needle in the same horizontal line, and over the needle in the plane of its meridian, then the deviation of the needle due to the rotation of the plate will be in contrary directions in the two cases, the plate revolving in the same direction in both. These and other peculiar effects arise

evening, and to have been decreasing from an early hour in the morning.

The general agreement of these intensities with those deduced from the observations made in doors, is as near as could be expected, considering that an interval of twenty days had elapsed between the two sets of observations. From this, and the agreement in the manner in which the westerly and easterly points of equilibrium approach and recede from the north in the two cases, which I have before pointed out, we may conclude, that there is nothing anomalous in the action which takes place on the needle under the different circumstances of its being placed in doors or in the open air; and that the apparent anomaly in the directions of the needle in the two cases, which was observed by Mr. BARLOW and myself, arose from the cause which I have assigned for it in my former paper; namely, the difference in the changes of temperature in the magnets when in doors and when in the open air.

The diurnal changes in the terrestrial magnetic intensity have been determined by Professor HANSTEEN, by means of the vibrations of a needle delicately suspended. From these observations it appears, that in general the time of minimum intensity was between ten and eleven o'clock in the morning; that the maximum happened between four and seven

entirely from the rotation of the iron, and are not produced by any friction on the axis. As the effects are not very considerable, to render them conspicuous it is necessary to make use of a plate eighteen inches in diameter, and to have its centre within sixteen inches of that of the needle. If the needle is under the influence of magnets, as in the foregoing observations, the effects produced by the rotation of the plate are considerable.

for the month of May 1820, and about seven o'clock in the evening for the month of June. The intensity which, in these observations, is taken as unity, is that deduced from an observation made during an aurora borealis; but for the purpose of comparison, I have, for the months of May and June, taken the intensity deduced from his observations at $10^h\ 30^m$ in the morning as unity, reduced the intensities, which he gives for other times in the day, to this standard, and placed them in the following table, with the corresponding intensities deduced from my own observations.

Intensity deduced from HANSTEEN's Observations in 1820.			Intensity deduced from the preceding Observations in 1823.		
Time.	May.	June.	Time.	May.	June.
h. m.			h. m.		
8 00 A. M.	1.00034	1.00010	7 30 A. M.	1.00114	1.00061
10 30	1.00000	1.00000	10 30	1.00000	1.00000
4 00 P. M.	1.00299	1.00251	4 30 P. M.	1.00175	1.00223
7 00	1.00294	1.00302	7 30	1.00220	1.00239
10 30	1.00191	1.00267	9 30	1.00231	1.00209

The principal difference to be observed in the nature of the changes of intensity during the day, in the two cases, is, that from my observations, the intensity appears to decrease more rapidly in the morning, and increase more slowly in the afternoon, than it does from those of Professor HANSTEEN; but the general character of these changes is as nearly the same as we can expect from methods so different, at different times, and at places where both the variation and dip of the needle are different. My object however was, to point out

what might be deduced from a series of such observations as I have detailed, rather than to compare the results deduced from them with those obtained by others, for which purpose it would have been necessary to have continued them for a greater length of time.

We have seen that with the magnets I made use of, their intensity being nearly 218 M, at the temperature 60°, a change in their temperature of 1° would cause a change of intensity of 0.123 M; or taking the intensity of the magnets 1, for each degree of increase in temperature we should have a decrease of intensity of 0.000564. Now if the same, or nearly the same, take place with all magnets, it is evidently necessary, in all cases where the terrestrial magnetic intensity is to be deduced from the vibrations of a needle, that great care should be taken to make the observations at the same temperature; or, the precise effect of change of temperature having been previously ascertained, to correct the observations according to the difference of the temperatures at which they were made. I am not aware that any one has yet attempted to make such a correction; but it is manifest from the experiments I have described, that it is indispensible, in order to deduce correct results from the times of vibration of a needle in different parts of the earth, where the temperatures at which the observations are made are almost necessarily different, that these temperatures should be registered, and the times of vibration reduced to a standard of temperature. It appears to me, that the effects will be the most sensible in large and powerful needles; and consequently, in making use of such, the reduction for a variation of temperature will

be most necessary. There would be no difficulty in this reduction, if we could give in terms of the intensity of any magnet the increment or decrement of intensity corresponding to a certain decrement or increment of temperature at all temperatures. To determine this accurately, would however require a great variety of experiments to be made with magnets of very different intensities; but as I have not made these, I must content myself for the present with pointing out some of the facts which I have ascertained from more extended experiments than those I have already given, reserving the detail of these experiments for another opportunity, should they be deemed of sufficient interest.

These experiments were made with a balance of torsion, the needle being suspended by a brass wire $\frac{1}{450}$ inch in diameter: by them I ascertained the following facts:

1. Commencing with a temperature — 3° FAHRENHEIT, up to a temperature 127°, as the temperature of the magnets increased, their intensity decreased. Owing to the almost total absence of snow during the winter, I was unable to reduce lower the temperature of the large magnets which I made use of; but from an experiment I made at the Royal Institution, in conjunction with Mr. FARADAY, in which a small magnet, enveloped in lint well moistened with sulphuret of carbon, was placed on the edges of a basin containing sulphuric acid, under the receiver of an air pump, I found that the intensity of the magnet increased to the lowest point to which the temperature was reduced, and that the intensity decreased on the admission of air into the receiver, and consequent increase of temperature in the magnet. This

is in direct contradiction to the notion which has been entertained of destroying the magnetism of the needle by the application of intense cold.

2. With a certain increment of temperature, the decrement of intensity is not constant at all temperatures, but increases as the temperature increases.

3. From a temperature of about 80° the intensity decreases very rapidly as the temperature increases: so that, if up to this temperature, the differences of the decrements are nearly constant, to ascertain which requires a precision in the experiments that perhaps their nature does not admit of, beyond this temperature, the differences of the decrements also increase.

4. Beyond the temperature of 100°, a portion of the power of the magnet is permanently destroyed.

5. On a change of temperature, the most considerable portion of the effect, on the intensity of the magnet, is produced instantaneously; showing that the magnetic power resides on or very near the surface. This is more particularly observable when the temperature of the magnet is increased, little change of intensity taking place after the first effect is produced; on the contrary, when the temperature of the magnet is diminished, although nearly the whole effect is produced instantly, yet the magnet appears to continue to gain a small power for some time.

6. The effects produced on unpolarized iron by changes of temperature are directly the reverse of those produced on a magnet; an increase of temperature causing an increase in the magnetic power of the iron, the limits between which I observed being 50° and 100°. That the effect on iron of an

increase of temperature should be the reverse of that produced on a magnet, is, I think, a strong argument against the hypothesis, that the action of iron upon the needle arises from the *polarity* which is communicated to it from the earth.

It may be objected to the method which I have adopted for determining the diurnal changes in the terrestrial magnetic intensity, that, after the observations have been made, they require a correction for temperature, which can only be determined by experiments previously made on the magnets and needle employed. The same objection may, however, be made against the method of determining the intensity by the vibrations of a needle. As such a correction has not in the latter case been hitherto applied, the results which have been obtained relative either to the diurnal changes of intensity, or the intensities in different parts of the earth, by means of observations on the vibrations of a needle, will be so far incorrect as the needle may happen to have been affected by differences in the temperature. The method I have described, however, possesses advantages over the other: a very considerable one is, that whatever effects are produced may easily be observed with considerable precision, the time required for each observation being not more than five minutes; another is, that, the magnets being immersed in water, as far as regards them, we may command the temperature at which the observations are to be made, and thus limit the correction for temperature to a very small quantity; and it possesses another decided advantage, that whatever are the effects produced on the needle by atmospheric changes, they are, by means of it, rendered immediately visible, and can be observed as they occur.

It was my intention to commence a series of such observations at the beginning of the present year, and to continue them for as long a period as I was able; but circumstances prevented my commencing at the time I proposed, and ill health has since put it out of my power to engage in such continued observations as would be required: but I trust the task will be undertaken by others who feel interested in investigating the phœnomena connected with terrestrial magnetism.

II. *The Croonian Lecture. On the existence of Nerves in the Placenta.* *By Sir* EVERARD HOME, *Bart. V. P. R. S.*

Read November 18, 1824.

IN the Lecture which I gave last year, I attempted to trace the structure of the human brain to as great a degree of minuteness as is consistent with accuracy, by observing its appearance in the field of the microscope. This I should not have ventured to do under any other circumstances, than being assisted by the eye of Mr. BAUER in examining the appearances, and in having correct representations of them under his hand, to lay before the Society.

Without these peculiar advantages, I should have been afraid of being led into error, either by the fallacies to which microscopical observations are liable in themselves, or those which so frequently occur when the same eye is not employed both in ascertaining the appearances, and in directing the pencil by which they are delineated.

As Mr. BAUER continues to indulge me with the same advantages, I shall employ them in the present Lecture in prosecuting my enquiry respecting the nerves; for as no anatomist before me has had the assistance of such an able coadjutor, it may never happen again; and I should feel myself undeserving of it, were I not to employ it in extending our researches in minute anatomy.

This I have now been enabled to do in no common degree,

by discovering nerves in both the fœtal and maternal portions of the placenta. This discovery, I am proud to say, was not the result of accident, but of a regularly arranged plan for that purpose.

In examining, at my desire, the structure of the horns of the fallow deer during their growth, while covered with velvet, Mr. BAUER found them abundantly supplied with nerves. The circumstance of nerves being met with, where sensibility is not only unnecessary, but even where the parts are unfitted for their office till the nerves are removed; which takes place as soon as the horns are full grown; makes it appear that nerves answer some other purposes in the animal œconomy, besides regulating the actions of the arteries; an office which, many years since, I not only considered them to perform, but illustrated my opinion by the effects of irritation on the parvagum and great sympathetic nerve on the carotid artery. Since that time, by considering the incubation of the chick, I have been led to believe that the arteries are indebted to the nerves for their formation; and so strong was the conviction on my mind of this being the case, that even the circumstance of the placenta, whose blood vessels are very numerous, having been suspected to have no nerves, did not induce me to abandon it; since until it is proved that the placenta is devoid of nerves, there is no argument against me. This was a point, of all others, that no one could so well determine as Mr. BAUER. I therefore most earnestly requested him to employ his microscopical observations on this subject, and supplied him with the placenta of a seal, in which the arteries and veins had been injected; and as in that animal the umbilical vessels are not

twisted, the nerves will be more exposed, and the parts having been in spirit, will have lost that transparency belonging to them in a recent state, by which they are less readily distinguished from blood vessels.

In this specimen, Mr. BAUER has shown that nerves are not only conspicuous surrounding the umbilical arteries, but has demonstrated them in the portion belonging to the uterus. For the appearance they put on, I must refer to the annexed drawing.

I may here remark, that in no communication which I have made to the Society, assisted by Mr. BAUER's labours, has any appearance been mentioned or represented, that I have not myself distinctly seen; for although I am not equal to the nice adjustment of the microscope, which indeed appears peculiar to Mr. BAUER, yet, when adjusted by him, the appearances before they were described had been rendered visible to me.

In looking at objects so much magnified beyond what they appear to the naked eye, it will not be unnatural for many of the Members to ask, how I am sure that these are really nerves, and not the secondary order of blood vessels, too small to carry red blood, and therefore, when their contents have been coagulated, appear to be chords? My answer to this question is, to recommend an inspection of the drawings; in which it will be seen, that these are not continuations of other branches, but form a trellis-work upon the arterial trunks, in a manner totally different from any thing met with either in the ramifications of arteries or veins; and when they are dried upon glass, they reflect the light with a degree of splendour like the human hairs when these

are quite white. When the nerves are very minutely examined, each fibre appears to consist of a row of small globules connected with one another.

At the time the nerves in the placenta were discovered, Sir STAMFORD RAFFLES (whose misfortune in having lost the most valuable collection in Natural History ever made in the East Indies by the ship taking fire, every one must feel for) brought me from Sumatra the pregnant uterus of the tapir of that country; and as in that animal the umbilical chord is connected with the chorion (there being no placenta), I examined the transparent portion of the chorion along which the branches of the funis pass, before they arrive at the spongy part, and there the nerves are so conspicuous, that Mr. BAUER's representation of them of the natural size is annexed.

The principal object of the present Lecture is to establish the fact of nerves existing in the placenta; and in these animals in which there is no placenta, in the floculent chorion, which is substituted for it; and it is a curious fact, that they should be largest in the latter.

This discovery places the placentular circulation in a new point of view, since, from the known influence of the nerves on the blood vessels, it is reasonable to believe that, during life, there are branches of communication between those of the uterus and fœtus, although too minute to be explored in the dead body. The erection of the penis cannot be produced after death by injecting the arteries, although when the nerves are excited the smaller branches give a ready passage to the blood. Having traced nerves from the fœtus to the maternal portion of the placenta, it will add to the value of this com-

munication, to give some general account of the course of the nerves which supply the uterus of the mother, more especially as these are little known in the different classes of animals, even to those who are well versed in comparative anatomy.

That some very important office is performed by the uterine nerves is evident from their number, the different sources from which they originate, and the various ganglia by which the filaments are connected with one another; and that such a complex system of nerves is required for the well doing of the fœtus in utero cannot be doubted, since they become enlarged during pregnancy. Mr. CÆSAR HAWKINS has very kindly made the dissections necessary for this purpose, and I shall give in his own words the account he has drawn up of the distribution of the nerves connected with the organs of generation of the female in the human species, in the quadruped, the bird, and the frog.

"The nerves of the human uterus are supplied from six different plexuses. The spermatic plexus within the abdomen, the great hypogastric plexus between the common iliac arteries, and four within the pelvis, two of which are situated on each side of the uterus. All of these have the peculiar appearance of the sympathetic nerves, and they are intimately connected with all the other nerves of the viscera.

"The uterine nerves in the dog, cat, rabbit, and guinea pig, so nearly resemble those of the human uterus, that a minute description of them is unnecessary. The spermatic plexus is formed by branches of the renal plexus and two nearest lumbar ganglia of the sympathetic nerve; it supplies the horns of the uterus, the ovaria, and apex of the urinary bladder.

" The common hypogastric plexus, after having supplied the body of the uterus, gives off a large nerve of considerable length, which dips down into the pelvis, and unites with numerous branches of the third sacral nerve, and smaller branches from the second and fourth; a remarkable plexus is thus formed which contains several distinct ganglia. It distributes nerves to the body of the uterus, the vagina, bladder, and rectum, the integuments of the upper part of the pubes, and the muscles of the inferior outlet of the pelvis. A few branches pass down to communicate with the fourth sacral nerve, where it gives origin to the pudic nerve. These nerves arise from the plexus in such a way as to resemble the ramifications of the venæ vorticosæ in the choroid membrane of the eye.

" The difference therefore between the nerves of the human uterus, and those belonging to the uterus in the quadruped, consists in the formation of only one lateral hypogastric plexus, and consequently in the existence of only four nervous centres in the latter. There appear also to be more ganglia in the plexiform distribution of the sympathetic nerve. In the seal, several large ganglia are found in the broad ligaments of the uterus.

" The nerves belonging to the female organs of birds are distributed as follows:

" The sympathetic nerve is found close to the origin of the spinal nerves, protected by the double heads of the ribs between which it runs. The spinal nerves that correspond to the lumbar and sacral nerves in quadrupeds emerge near each other, and as the sympathetic nerve communicates with each of them, and forms a ganglion immediately after their

appearance from the vertebral foramina, there is an almost uninterrupted ganglion of considerable length; from which numerous filiments go off to supply the oviducts. Others run upwards and are distributed on the ovaria.

"Near the termination of the oviduct in the cloaca a plexus is formed, nearly similar to the lateral hypogastric plexus in quadrupeds, which is distributed in a corresponding manner to the oviduct and cloaca. There is also a similar pudental nerve.

"Fewer ganglia are formed near the aorta than in quadrupeds, and scarcely any branches are sent from the common hypogastric plexus to the oviduct.

"In the frog, as there is no proper sympathetic nerve, the abdominal viscera are supplied directly from the spinal nerves. These soon after they emerge from the vertebral canal, become slightly enlarged: this does not deserve to be called a ganglion. From each of the spinal nerves in the lower part of the back and loins, a small nerve is given off, which takes a direction towards the centre of the bodies of the vertebræ, where they unite with each other, and with the corresponding nerves of the opposite side. By this union a flat nervous web is formed, which stretches across the aorta and extends downwards into the pelvis: this is analogous to the splanchnic plexus in hot blooded animals.

"From the upper surface of this plexus many branches run upwards towards the intestines and kidneys, but the greater number are distributed on the ovaria.

"The lumbar nerves on each side give off several branches, which pass at once into the oviduct. The last lumbar nerves pass down upon the surface of the psoæ muscles, and near

the pubes give off a branch, which takes a circuitous course towards the lateral portion of the bladder, and the extremity of the oviduct. The continuation of the aortic, or abdominal plexus, in union with some branches of the sacral nerves, forms on each side of the pelvis a kind of plexus, which distributes branches to the cloaca and lower portion of the oviducts.

"The nerves corresponding with those which have been described in the frog, run almost entirely in straight lines, instead of having the intricate reticulated texture of the visceral nerves in hot blooded animals. The ganglia are indistinct, and the fibres that compose them resemble those of the muscular, more than visceral nerves."

At the time I was appointed to give this Lecture, I had completed an investigation, in which was traced to its origin the formation of the brain and spinal marrow in the ovum of the frog, and intended, upon this occasion, to have laid my observations, illustrated by a series of drawings made by Mr. Bauer, before the Society, but having been so fortunate as to discover the nerves of the placenta, I did not hesitate in giving this discovery the preference, and taking the earliest possible opportunity of communicating it to the Society.

Now that it is known by the discovery of the nerves in the placenta that the brain of the child, as well as every part of its body, is connected by the medium of nerves with the brain of the mother, we are led to understand the degree of dependence in which the fœtus is kept during the whole time of utero gestation.

The small pox being in some instances communicated from the mother to the child, which has until now been consi-

dered as an extraordinary fact* and not to be accounted for, is readily explained, since absorption depends fully as much upon nervous influence as the action of the arteries: a child in utero having an ague, is in itself almost a proof of the placenta having nerves.†

A child being born without a brain is not to be marvelled at, the nerves of the child being connected with the brain of the mother.

The immediate division of the navel-string at the moment of the birth, in some particular instances having hazarded the life of the child, hinted at by Dr. DENMAN, shows the accuracy of his observations.

* There are several cases in the Philosophical Transactions of children having the small-pox in utero, and one that was read before the Society, but not published.

Two of these were in England; in both of them the child took the infection on or about the 14th day.

One was in Jamaica; and the infection was taken by the child on the 8th day. This difference appears to deserve being recorded.

† Dr. PATRICK RUSSEL states a case of ague occurring in a child in utero, in Aleppo.

In June, 1767, a healthy young woman, in the seventh month of her third pregnancy, was attacked by a tertian fever, and the fœtus in utero appeared to suffer a paroxysm distinct from the mother.

The fits in the mother returned regularly about noon, and terminated by a profuse sweat in less than ten hours.

About 8 in the morning of the odd days, the woman felt the child tremble with great violence; she also felt a weight and coldness in the womb; the coldness went off in less than 15 minutes, and was succeeded for more than an hour by a glowing heat; the child was at intervals restless, as she had felt in her other children during pregnancy, but the trembling she never before experienced.

After the sixth paroxysm the bark effected a cure.

Dr. RUSSEL, while at Aleppo, met with a few similar instances, but had attributed them to the effect of imagination, which in this woman he could not do, as she was remarkable for her cheerful disposition, and good sense. *Trans. of a Society for the Improvement of Medical and Chirurgical Knowledge*, vol. iii. p. 96.

Till this discovery was made, we had no mode of estimating the influence that could be produced upon the child by the affections of the mind, or the body of the mother; and therefore the instances that have occurred, were considered as idle stories or accidental occurrences, for which no satisfactory reason could be assigned; and this upon no better ground than that they do not always take place under similar circumstances, which nothing connected with nerves ever does.

That they do sometimes occur no one can be so hardy as to deny, and when they do, we cannot now be at a loss for a mode of accounting for their doing so.

With the following well authenticated instances of this kind that have come under my own observation, I shall close this Lecture.

The mare that in the first instance had a foal by a quagga, and afterwards three in succession by a Persian horse, all of which were marked like the quagga, can in no other way be explained than through the influence of the mother upon its young. In proof of the fact, paintings from life of all the individual animals have a place in the Royal College of Surgeons in London. There is also an account of the occurrence stated in the Philosophical Transactions, in which all the particulars are detailed communicated by the Earl of Morton, to whom in the first instance the mare belonged.

A lady while pregnant with her tenth child, the former nine being perfectly formed, was robbed in the dusk of the evening near Woolwich by an artillery man who had a hare lip the fright occasioned by the alarm was so great: that she did not recover from it for several days; when brought to bed the child was found to have a hare-lip. I was called

upon to perform the operation. As the mother was very irritable and nervous, and the child unusually fractious, I used every means of soothing the child previous to the operation; but after it was performed, the child never ceased from crying for three days, and died at the end of that period.

A lady of an unusually nervous habit, who had several healthy children perfectly well formed, during a state of pregnancy, was opening the hall door of her house in the country, when a Newfoundland dog rushed upon her, and jumped up in play, putting his two fore paws upon her sides: the alarm and surprise was very great, but she soon recovered herself: when the child was born there was a claret mark upon the two parts of the belly that corresponded with the places on which the dog's paws had been placed. These I afterwards removed, and the parts readily recovered.

An Italian woman, twenty years of age, when by her reckoning three months and three weeks gone with her third child, was travelling in a caravan with the baggage of the Duke of Wellington's army, in the middle of the night, in a violent storm, while she was fast asleep, a small monkey with a long chain upon the roof of the caravan took refuge in it, crept under her loins, and fell asleep; she awoke, feeling uneasy from the pressure of the monkey, and put her hand down to scratch the part, but came upon the monkey's head, by which it awoke and bit her fingers, and in its alarm got fast hold of her loins. The woman went into fits, and was some minutes before she recovered herself: it was expected she would miscarry, but she went her full time. When the child was born it measured between seven and eight inches in height, and weighed one pound. This was in France. The child

was reared with great difficulty; it was carried afterwards to Ireland, and there was afflicted with a hacking cough; it was brought to England, in expectation of the cough being relieved, but died soon after, just before it completed its ninth year. I saw it before its death; at that time it was much emaciated, and measured exactly twenty-two inches. After death it was found that the fontinelle of the head had closed; no fat was any where met with but at the bottom of the orbits. The uterus was as small as in a fœtus between three and four months, not being at all developed, whereas in a new-born child it has acquired a considerable size; it was closely attached to the posterior surface of the urinary bladder, apparently by inflammation: the bladder was distended with urine, and the size of a turkey's egg. As the child had never passed its water freely from the time of its birth, this affection of the bladder must have taken place at the same time with the injury produced by the monkey's gripe upon the loins of the mother.

I examined an abortion, which was considered by Mr. CLARKE, Teacher in Midwifery, to be about three months and a half after impregnation; and on comparing the ovaria with those of the dwarf, they were nearly of the same size, but not quite so long; the difference however was scarcely observable.

The child when I saw it could walk alone, but not more confident in itself, or firm on its legs, than an healthy infant at sixteen months. Its sight was very quick, particularly in seeing bright objects; was delighted with every thing showy, much pleased with ornaments in its own dress, could speak in a very low tone and shrill voice, and had some taste for

music, but had few English words; appeared very sensible of kindness, and remembered perfectly those from whom it had received attention.

The mother has had a fifth child in Ireland, which, like her former children, was of the common size when born, and has nothing particular in its appearance.*

EXPLANATION OF THE PLATES.

PLATE II.

Fig. 1. A small portion of the placenta of the seal, exposing the chorion by which it is covered, through which are seen the arterial and venal branches injected with wax, magnified four diameters.

The folds of the chorion contain the branches of the nerves.

The cut edge of this portion exposes the structure of the placenta, which is more distinctly seen in Fig. 4.

Fig. 2. The same portion as Fig. 1. exposing the uterine surface, which appears to be a tissue of arteries, veins, and nerves, enveloped in a soft spongy coagulable lymph; magnified in the same degree as Fig. 1.

Fig. 3. A small portion of Fig. 2. magnified ten diameters, to show the tissue, and the parts of which it is composed

* Since this Lecture was read, two cases have been stated to me, in which I have the most implicit confidence. In one, a Lady in early pregnancy was frightened by a sailor with one arm, and her child was born under the same deformity. In another, this occurred late in pregnancy. No effect was produced on that child; but in her next child, although every alarm in her mind had subsided, the deformity was found to have taken place.

more distinctly; the broken off portions of nerves are shown projecting beyond the edges of the outline of the figure.

Fig. 4. A transverse section of the placenta magnified ten diameters, showing its structure: the nerves are so readily distinguished by their course from the blood vessels, as to require no explanation.

Fig. 5 and 6. A single floculus of the tissue separated, and magnified ten diameters, exposing the terminal branch of the umbilical artery (and its accompanying nerves), where it ends in pencilli of infinitely small ramifications.

A. The pencilli.

B. Surface of the chorion.

Fig. 7. The mode in which the arteries of the umbilical chord begin to ramify on the chorion, and dip down into the substances of the placenta, magnified two diameters; the nervous filaments are distinctly seen.

Plate III.

Fig. 1. A portion of the uterine surface of the chorion of the tapir; natural size.

Fig. 2. A very small portion of the same, magnified fifty diameters.

Fig. 3. Lateral view of a section of the same; magnified fifty diameters.

Fig. 4. A very small portion of the fœtal surface; magnified fifty diameters.

POSTSCRIPT.

Mr. BAUER and myself at the same time, unknown to each other, having detected nerves in the human navel-string and placenta, I beg to communicate this discovery to the Royal Society, as a valuable addition to the Croonian Lecture, to which I wish it to be annexed as a Postscript.

Mr. BAUER's delineation of these nerves makes it unnecessary to give any verbal description of them. They were found in a specimen belonging to Mr. BROOKS, which had been successfully injected, and preserved in spirit for forty years, which I borrowed for that purpose.

PLATE IV.

Fig. 1. A portion of the umbilical cord of the human placenta, in its natural form; magnified four diameters.

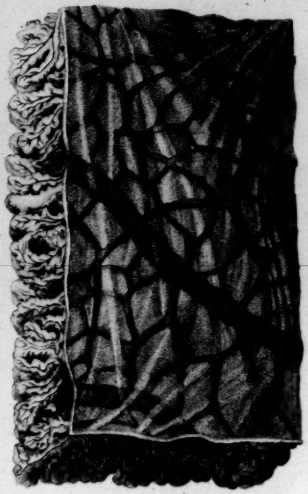
Fig. 2. The same portion unravelled, to show the situation of the nerves within it; magnified four diameters.

Fig. 3. A portion of the external membrane of the umbilical cord, with part of the cellular substance, and a nerve adhering to it; magnified four diameters.

Fig. 4. The same nerve shown separately; magnified ten diameters.

Fig. 5. A portion of the amnion, with a double branch of a nerve passing across it; magnified four diameters.

Fig. 1.



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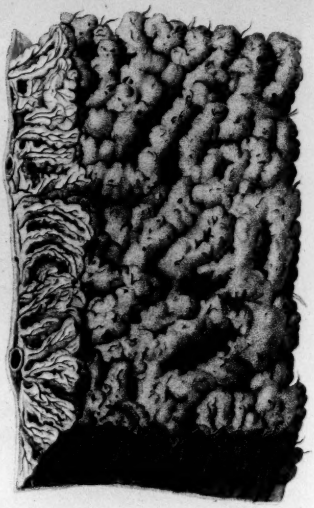

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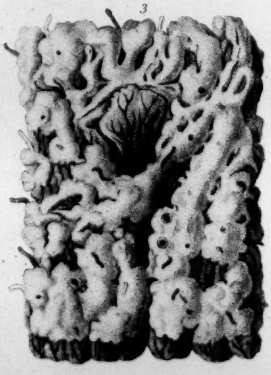

Fig. 1.

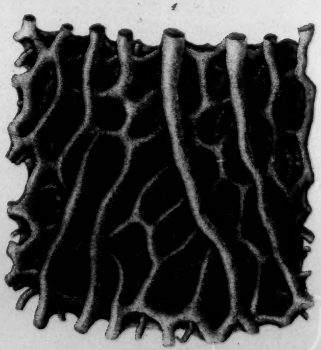

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Fig. 1

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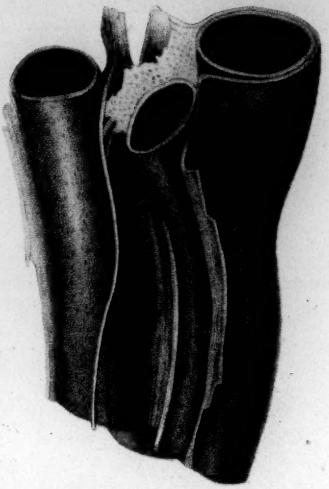

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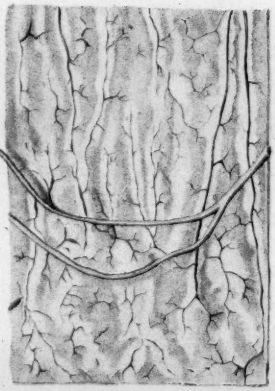


III. *Observations on the changes the Ovum of the Frog undergoes during the formation of the Tadpole.* By Sir EVERARD HOME, *Bart. V. P. R. S.*

Read November 25, 1824.

IN the year 1822, I laid before the Society a series of observations on the progress of the formation of the chick in the egg of the pullet, illustrated by drawings from the pencil of Mr. BAUER, showing that in the ova of hot-blooded animals the first parts formed are the brain and spinal marrow. I have now brought forward a similar series on the progress of organization in the ova of cold-blooded animals, illustrated in the same manner by microscopical drawings made by the same hand.

By comparing together the first rudiments of organization in the ova of these very distinct classes of animals, I shall be able to prove that, in both, the same general principle is employed in the formation of the embryo.

This enquiry has its interest considerably encreased, by the ova not being composed of similar parts.

The ova of the frog, which have been selected for this investigation, are found to have no yelk. If we examine these ova in the ovaria in which they are formed, we find them to consist of small vesicles of a dark colour; when they enter the oviducts they enlarge in size, and acquire a gelatinous covering, which increases in quantity in their course along those tubes; but the ova can neither be said to have acquired

their full size, nor to have received their proportion of jelly, till they arrive at a cavity close to the termination of each oviduct, formed by a very considerable enlargement of those tubes, corresponding, in many respects to the cloaca in which the pullet's egg is retained till the shell becomes hard.

These large bags, in which the oviducts of the frog terminate, when distended with ova, put on an appearance so like the enlarged horns of the uterus of the quadruped when they are filled with young ones, that they have by some anatomists been called a double uterus. This, however, is an improper appellation.

When the ova are deposited in these reservoirs, they become completely formed, and in a state to be impregnated by the male influence, which is applied to them in the act of their expulsion. As they are pressed upon each other, by being confined in a small space, the gelatinous covering takes on an hexagonal figure, in the centre of which is the ovum.

The ova, when examined by a magnifying glass in a strong light, exhibit an appearance so similar to the molecule in the pullet's egg, as to be readily mistaken for it; but a more attentive inspection shows, that it is only a white portion in the ovum, seen through the covering of the vesicle. When the vesicle is punctured by the point of a needle, the contents are so fluid as readily to run out, leaving the strong transparent membranous bag lined with a fluid nigrum pigmentum, empty.

Immediately after impregnation there is no change in the appearance of the jelly, nor of the vesicle contained in it, in

this respect corresponding exactly with what happens to the pullet's egg. The first change that is produced towards the formation of an embryo is, the contents of the vesicle expand, its form changes from that of a sphere to an oval, and when cut through its contents are no longer fluid. In the act of coagulation, the central portion becomes of a lighter colour than that which surrounds it, swells out in the middle, and there is a distinct line by which the two portions are separated from one another: the central part, in its future changes, is converted into brain and spinal marrow, and after these organs have acquired a defined outline, the heart and other viscera are seen forming in the darker substance.

This does not exactly correspond with what takes place in the pullet's egg, that of the frog having no yelk. In the pullet's egg, the part within the inner circle of the molecule, when impregnated by the male, undergoes the necessary changes to form the brain and spinal marrow; the part within the outer circle forms the blood and its vessels; the supplies out of which the other organs are to be produced, are afterwards derived from the yelk.

The membrane that forms the vesicle which is destined to contain the embryo when it has become a tadpole, has a power of enlargement as the embryo increases in size, and then performs the office both of the shell and of the membrane that lines it in the pullet's egg, at the same time serving as a defence to protect it, and allow of the blood being aerated.

The nigrum pigmentum lining the vescicle can only answer some secondary purpose, since it is not met with in the aquatic salamander, whose mode of breeding very closely

resembles that of the frog. Upon reflecting that the frog's spawn is exposed to the scorching effect of the sun, and in places where there is no shelter, this nigrum pigmentum may be given to the eggs as a defence for the young during its growth, which cannot be required in those of the aquatic salamander, since they are separately inclosed within the twisted leaves of water plants, and screened from the full force of the sun's rays. The plant whose leaves the aquatic salamander most generally selects to lay its eggs upon is the Polygonum persicare.

EXPLANATION OF THE PLATES.

PLATE V.

Fig. 1. A female frog laid open, just ready to shed her spawn; natural size.

Fig. 2. The ovaria and oviducts; natural size.

Fig. 3. Ova of different sizes taken from the ovarium; magnified five diameters.

Fig. 4. Ova from the upper part of the oviduct; magnified five diameters.

Fig. 5. Ova from the dilated portion of oviduct; natural size. A B. Two ova that had been a few minutes in water, to show the expansion of jelly; magnified five diameters.

Fig. 6. An ovum from which the jelly is removed; magnified ten diameters.

Fig. 7. The same ovum opened, to show that the contents are fluid; when they are allowed to coagulate and dried upon glass, ramifications are formed as in coagulating blood; magnified ten diameters.

Fig. 8. Some ova after being immersed in water for fourteen days; natural size.

Fig. 9. One of them magnified five diameter.

Fig. 10. The same ovum magnified ten diameters.

Fig. 11. The same ovum opened; its contents still fluid, in which oil is found; magnified ten diameters.

Plate VI.

Fig. 1. Ova six hours after being spawned on water; natural size. A. one of these ova; magnified five diameters. B. The same ovum magnified ten diameters. C. Longitudinal section of the same ovum; its contents in a half coagulated state, and putting on an organized structure; magnified ten diameters.

The rest of the ova spawned at the same time were kept in water, to watch the progress of the formation of the tadpole.

One of these was not impregnated, consequently remained unchanged, while the others became gradually more and more organized. This abortive ovum is placed at the top of each cluster with a mark *

Fig. 2. The cluster of ova 12 hours after being spawned, diminished by that examined in fig. 1.; of the natural size. A. One magnified five diameters. B. The same magnified ten diameters. C. The same, forming a longitudinal section; magnified ten diameters.

Fig. 3. Twenty-four hours after being spawned.

Fig. 4, Thirty-six hours after being spawned. At this period the ovum has its form considerably changed, and the head and tail of the tadpole are distinctly seen.

Fig. 5. Three days after being spawned. At this period muscular motion is for the first time perceptible. The letter D shows the ovum laid open on one side; magnified ten times, as in letter C.

Fig. 6. Four days after being spawned. The letters correspond to these in Fig. 5.

PLATE VII.

Fig. 1. Five days after being spawned. At this period the ova become separated, and the tadpoles begin to leave the ovum. A. Shows a tadpole in the act of extricating itself; magnified five diameters.

B. A back view of it; magnified eight diameters.

C. A belly view.

D. A side view.

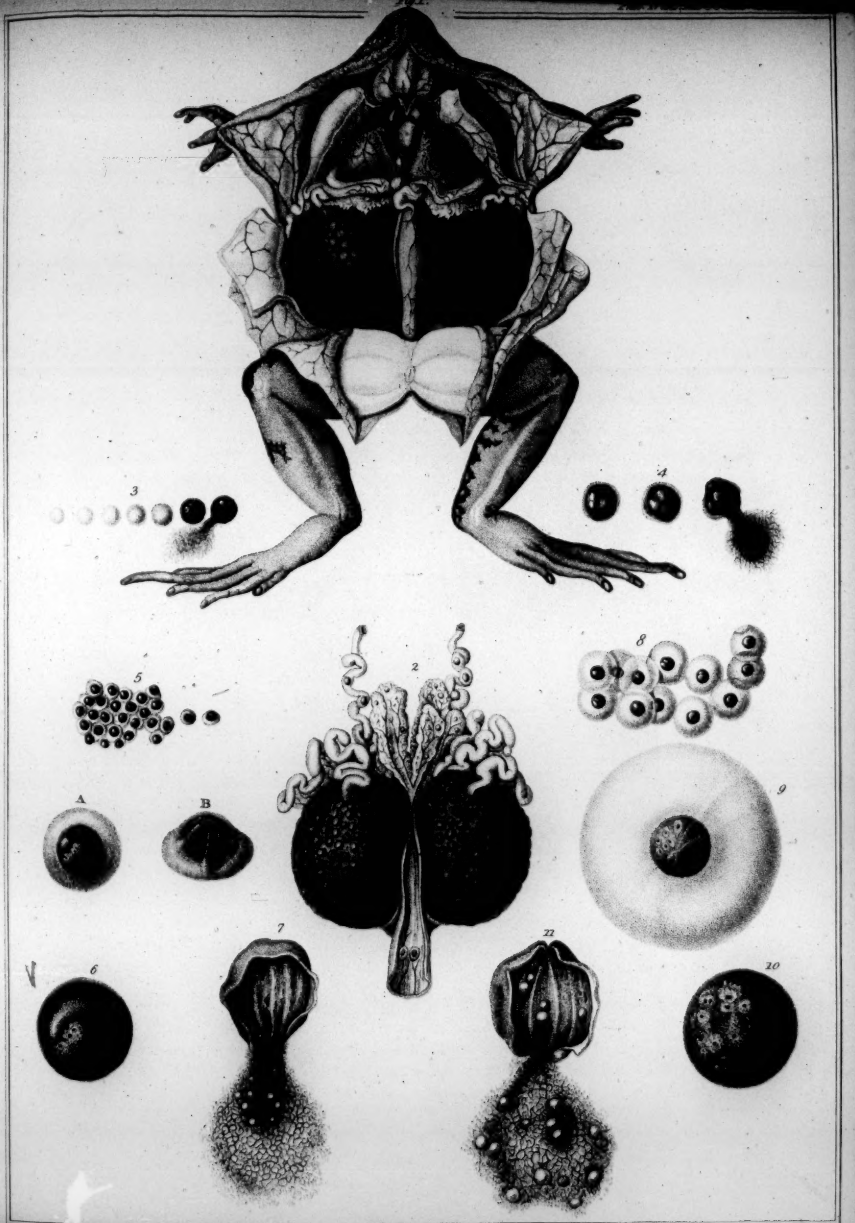
E. A longitudinal section; all the views magnified eight diameters.

Fig. 2. Six days after being spawned, four different views; magnified eight diameters, as in Fig. 1.

Fig. 3. Eight days after being spawned; all the views magnified eight diameters.

Fig. 4. Twelve days after being spawned; four views of the tadpole; magnified eight diameters.

The animal in twelve days had become so far advanced in its growth to make further progress in the investigation unnecessary, after the splendid figures that are before the public upon that subject in different publications.



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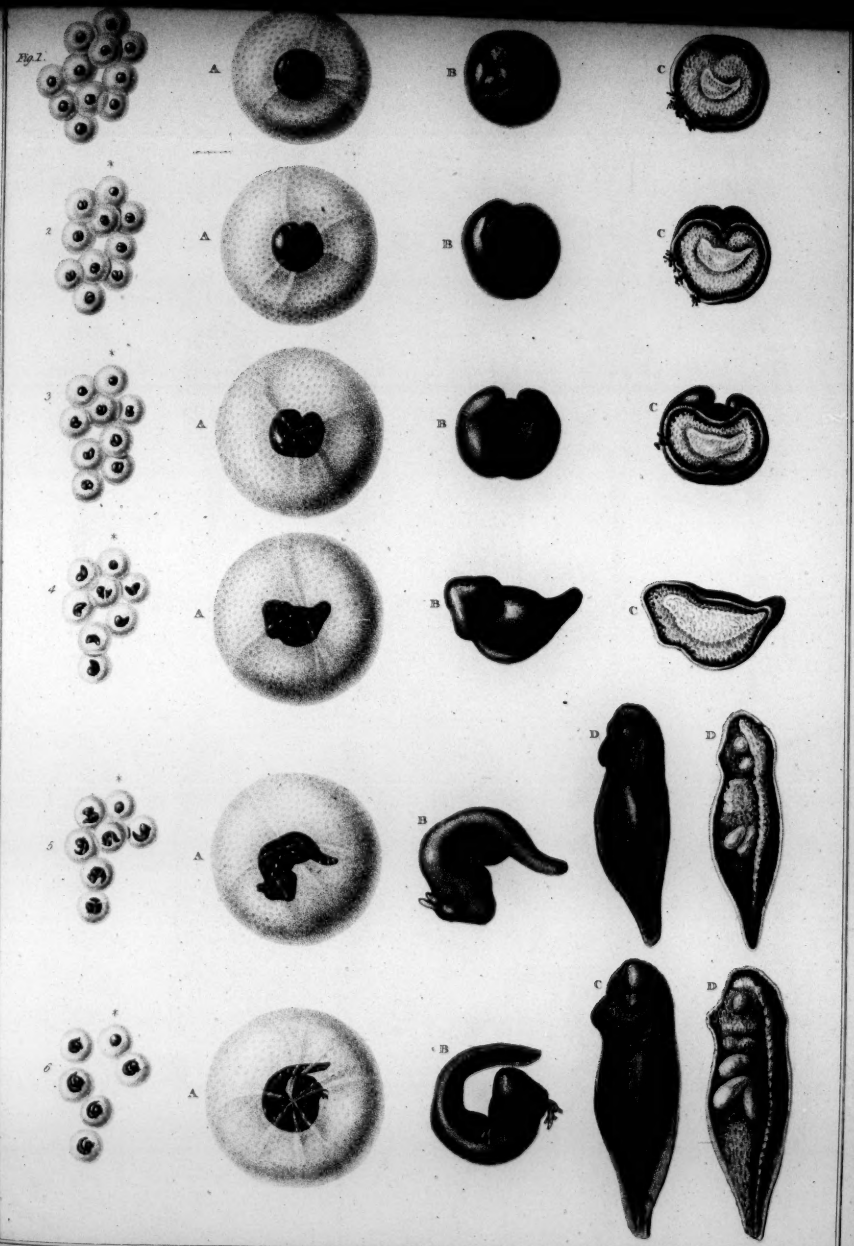

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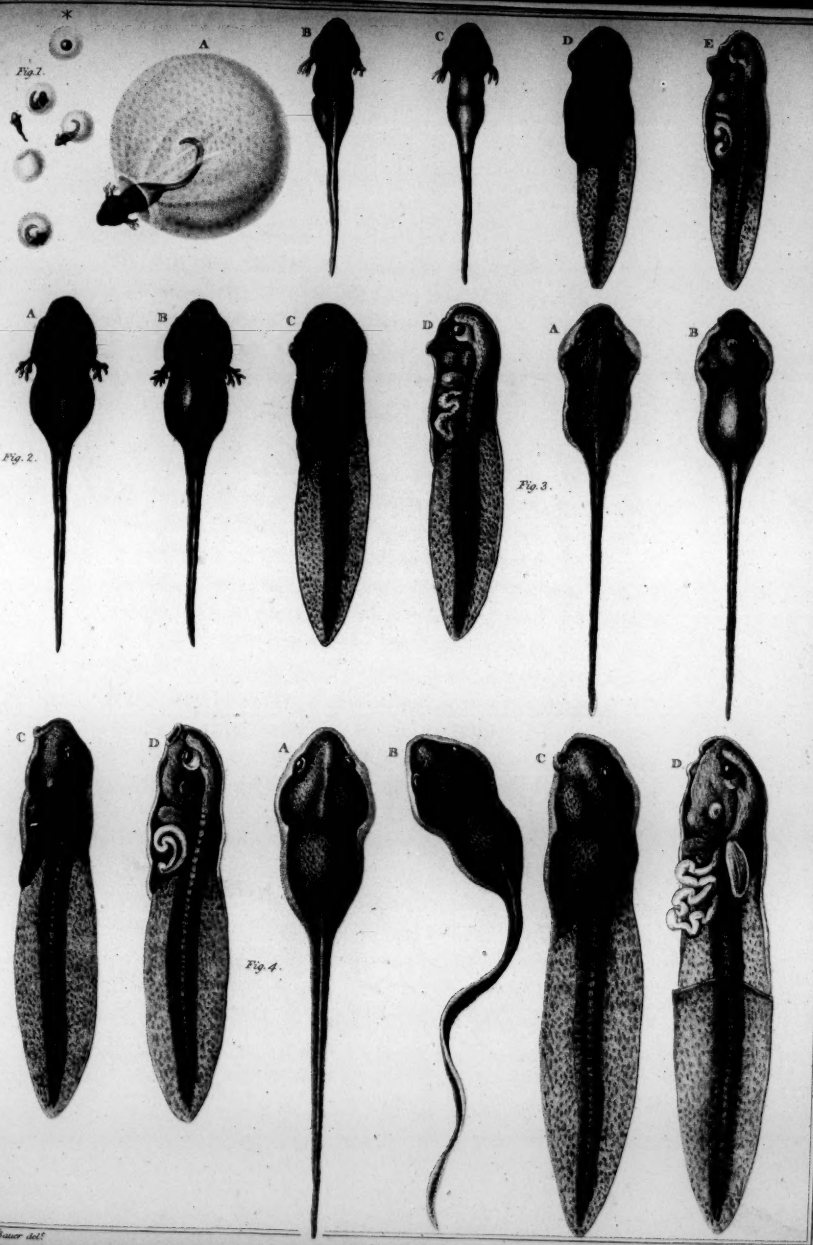

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IV. *A general Method of Calculating the Angles made by any Planes of Crystals, and the Laws according to which they are formed. By the Rev.* W. WHEWELL, *F. R. S. Fellow of Trinity College, Cambridge.*

Read November 25, 1824.

1. IT has been usual to calculate the angles of crystals and their laws of decrement from one another, by methods which were different as the figure was differently related to its nucleus; which were consequently incapable of any general expression or investigation, and which had no connexion with the notation by which the planes of the crystals were sometimes expressed. And the notation which has hitherto been employed, besides being merely a mode of registering the laws of decrement, without leading to any consequences, is in itself very inelegant and imperfect. The different modes of decrement are expressed by means of different arbitrary symbols; and these are combined in a manner which in some cases, as for instance in that of intermediary decrements, is quite devoid both of simplicity and of uniformity, and indeed, it may be added, of precision. The object of the present paper is to propose a system which seems exempt from these inconveniences, and adapted to reduce the mathematical portion of crystallography to a small number of simple formulæ of universal application. According to the method here explained, each plane of a

crystal is represented by a symbol indicative of the laws from which it results; the symbol, by varying the indices only, may be made to represent any law whatever: and by means of these indices, and of the primary angles of the substance, we obtain a general formula, expressing the dihedral angle contained between any *one plane* resulting from crystalline laws, and any *other*. In the same manner we can find the angle contained between any *two edges* of the derived crystal. Conversely, knowing the plane or dihedral angles of any crystal, and its primary form, we can by a direct and general process deduce the laws of decrement according to which it is constituted. The same formula are capable of being applied to the investigation of a great variety of properties of crystals of various kinds, as will be shown in the sequel. We shall begin with the consideration of the rhomboid, and the figures deduced from it; and we shall afterwards proceed to other primary forms.

§ 1. *The Rhomboid.*

2. Let there be a rhomboid, A *a*, Fig. 1. divided into a number of small equal rhomboids by planes parallel to its faces. Let any one of the points of division of each of its three upper edges be taken, as P, Q, R; and let a plane pass through these three points P, Q, R. Let the small rhomboids which are above this plane be removed, so as to leave a uniform assemblage of cavities. Then, the remaining surface P Q R, being composed of the trihedral angles of small rhomboids, if we suppose the small rhomboids to become smaller than the least distinguishable magnitude, the surface P Q R will appear a plane. And if we suppose these rhomboids to represent the

primary form of a crystalline body, P Q R will be a secondary surface deduced from a certain arrangement of these primary elements.

Let the three upper edges of the rhomboid, A x, A y, A z, be considered as three axes of co-ordinates; and let the corresponding co-ordinates be x, y, z. We can then express the plane P Q R by means of these co-ordinates. If, for instance, we consider an edge of the small rhomboid as unity, and if AP, AQ, AR contain respectively 9, 6, and 3 of these edges, the equation to the plane P, Q, R, will be

$$\frac{x}{9} + \frac{y}{6} + \frac{z}{3} = 1:$$

and if the numbers of small rhomboids in AP, AQ, AR be respectively h, k, l, the equation to the plane will be

$$\frac{x}{h} + \frac{y}{k} + \frac{z}{l} = 1.$$

If h, k, l be multiplied by any common quantity m, so that the equation becomes

$$\frac{x}{mh} + \frac{y}{mk} + \frac{z}{ml} = 1, \text{ or } \frac{x}{h} + \frac{y}{k} + \frac{z}{l} = m,$$

it is clear that the plane P Q R will continue parallel to its former position, and may be considered as deduced from the same law as before. Hence it appears, that in the equation $\frac{x}{h} + \frac{y}{k} + \frac{z}{l} = m$, the quantity m does not serve to determine the position or law of formation of the plane, and may be any whatever. If we make $m = 0$, the plane P Q R, still continuing parallel to its former position, will pass through the point A; and as we have to consider only the *angles* made by planes and their intersections, we may in such calculations suppose all our planes to pass through this point A.

Since therefore the direction of the plane P Q R is completely determined by the three quantities h, k, l, we may represent it by writing those three quantities thus $\left(\frac{1}{h}; \frac{1}{k}; \frac{1}{l}\right)$;* or, if the equation be $px + qy + rz = m$, we may represent the plane by the symbol $(p; q; r)$.

3. According to the law of symmetry which prevails in the production of crystalline forms, if one edge or face of the primary solid be modified in any manner, the other homologous edges and faces will be similarly modified. Hence, if one plane exist, other corresponding planes must also exist, and these we may call *co-existent planes* to the first.

Thus if we have a plane P Q R, Fig. 2, and if we take $AP' = AQ$, and $AQ' = AP$, we must also have a plane $P'Q'R$: for the edges Az, Ay being perfectly similarly situated, if one of them be affected in any manner, the other must be similarly affected. Hence, if we have a plane $(p; q; r)$, we must have one $(q; p; r)$. The same is also true of z; and by considering this in the same manner, it will be seen that the plane $(p; q; r)$ has the following co-existent planes

$$(q; p; r)\ (r; q; p)\ (p; r; q)\ (q; r; p)\ (r; p; q).$$

That is, there are all the permutations that can be made by altering the arrangement of the three quantities p, q, r; that the one which stands first in order being always the coefficient of x, the second that of y, and the third that of z.

These six planes may be represented by a single symbol

* We might represent the plane by $(h; k; l)$, which shows more immediately the law of its formation; but in all our subsequent calculations we have to use the reciprocals, and hence our formulæ are simplified by using the symbol $(p; q; r)$ where p, q, r are the coefficients of the equation.

(p, q, r); it being understood, that when quantities are only separated by *commas*, they are to be taken in all the ways in which they can be permuted. In the same manner $(p, q; r)$ may represent the two planes $(p, q; r)$ $(q, p; r)$, the permutations not extending to r, which is separated by a *semicolon*. In the case of the rhomboid, however, the permutations always include all the three quantities, in consequence of the similarity of its three edges.

4. We have hitherto considered only the planes produced by cutting off the upper angle; but we may represent in the same manner the plane produced by truncating any other angle. It may be observed that the angles x, y, z, fig. 3, which are separated from the *superior angle* A by an edge, are called *lateral angles*. The angles x', y', z', which are separated from A by a diagonal, are called *inferior angles*.

Let $p\,q\,r$, fig. 3, be a plane produced by a truncation at the lateral angles: xp, xq, xr being h, k, l respectively. Produce rA beyond A, and take AP $= xp$, AQ $= xq$, AR $= xr$; then the plane P Q R will be parallel to $p\,q\,r$, and may be taken instead of it. Now it is manifest that the equation to this plane is

$$-\frac{x}{h} + \frac{y}{k} + \frac{z}{l} = 1;$$

and therefore its symbol is $\left(-\frac{1}{h}; \frac{1}{k}; \frac{1}{l}\right)$. Or if $p = \frac{1}{h}$, $q = \frac{1}{k}$, $r = \frac{1}{l}$, the equation is $-p\,r + qy + rz = m$, and the symbol $(-p; q; r)$. Hence a plane which cuts off the lateral solid angles is distinguished by having one negative index.

In the same manner let $p\,q\,r$, fig. 4, cut off an inferior angle x', so that $x'p = h$, $x'q = k$, $x'r = l$: and taking

AP $= x'p$, AQ $= x'q$, AR $= x'r$, the plane PQR will be parallel to $p\,q\,r$, and its equation will be

$$\frac{x}{h} - \frac{y}{k} - \frac{z}{l} = 1 \text{ ; or } px - qy - rz = 1 :$$

and its symbol $\left(\frac{1}{h}; -\frac{1}{k}; -\frac{1}{l}\right)$, or $(p; -q; -r;)$. Hence a plane which cuts off the inferior solid angles is distinguished by having two negative indices.

It may be observed, that in both these cases the *coexistent* planes are given by taking the permutations of p, q, r; and may be represented as before by $(-p, q, r)$ and $(p, -q; -r)$. There will in each case be six; two for each angle.

5. If one of the quantities AP, AQ, AR, or h, k, l, in any of these cases become infinite, we shall have a truncation of an *edge* of the rhomboid. Thus if AP, in fig. 2, become infinite, we have a plane cutting off the terminal edge Ax, fig. 5. And since h is infinite, if $q = \frac{1}{k}$, $r = \frac{1}{l}$, the equation of this plane is $qy + rz = 1$; and its symbol $(0; q; r)$.

In the same manner, making $x'r$ infinite in fig. 4, we have, for a plane truncating the lateral edge $x'y$, an equation $px - qy = 1$, and a symbol $(p; -q; 0)$.

The terminal edges of Ax, Ay, Az, are not similarly affected with the lateral edges xy', $y'z$, zx', $x'y$, yz', $z'x$.

6. Instead of supposing the secondary faces to be produced by removing a part of the rhomboid Aa, we may conceive, with HAUY, that this larger figure is composed by adding successive layers of the small component rhomboids to a rhomboidal nucleus; and that the secondary faces are produced by supposing the magnitude of these layers to decrease according to any law. And it will be easy to show

what symbols, according to the notation here proposed, correspond to the different laws in the old system. Thus

A decrement on the superior angle is expressed by (p, q, q), which corresponds to HAUY's symbol $\frac{A}{\frac{p}{q}}$.

On a lateral angle by $(-p, q, q)$ corresponding to $E^{\frac{q}{p}}$;

On an inferior angle by $(p, -q, -q)$ corresponding to $e^{\frac{q}{p}}$;

On a terminal edge by (o, q, r) corresponding to $B^{\frac{r}{q}}$;

On a lateral edge by $(p, -q, o)$ corresponding to $G^{\frac{q}{p}}$.

An intermediary decrement thus (p, q, r), corresponding to $\left(A^p B^q C\frac{q}{r}\right)$ and $(p, -q, -r)$ corresponding to $\left(O^p D^q F\frac{q}{r}\right)$.

The symbols of the faces of the primary form are (p, o, o).

7. There is in fact, however, no necessity to suppose the secondary forms to be produced either by truncation of a primary one, or by addition to it. If we suppose that the small rhomboids, of which A *a* was assumed to be made up, are continued through all the space round the point A, we may conceive a plane to pass among these, parallel to P Q R. And this plane will be represented by $(p\,;\,q\,;\,r)$ independently of any consideration of the rhomboid A *a* or the point A; for if we take *any point*, and from it draw lines to the plane, parallel to the three edges A *x*, A *y*, A *z*, these three lines will be as $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$. And any other plane may similarly pass among the small rhomboids, and be represented by $(p'\,;\,q'\,;\,r')$. And if we obtain any solid figure contained by such planes, we may, by supposing those of the small

rhomboids which lie without this plane to be removed, have a proper representation of a secondary crystalline form constituted by the aggregation of primary ones.

Before we proceed to the calculations founded on this mode of viewing the subject, we may observe, that by increasing or diminishing the three indices p, q, r in any ratio, the plane represented by them is not altered. Thus $(p\,;\,q\,;\,r)$ $(np\,;\,nq\,;\,nr)$ $\left(\frac{p}{r}\,;\,\frac{q}{r}\,;\,1\right)$, &c. are the same plane. Hence $(p\,;\,q\,;\,q)$ is the same as $\left(\frac{p}{q}\,;\,1\,;\,1\right)$ $(p\,;\,p\,;\,0)$ as $(1\,;\,1\,;\,0)$; and the primary faces are $(1, 0, 0)$.

8. PROP. To find the dihedral angle contained between two planes $(p\,;q\,;r)$ $(p'\,;\,q'\,;\,r')$, the dihedral angle at the terminal edges of the primary rhomboid being α.

If there be three co-ordinates any how situated so that the dihedral angle at the axis x between the planes xy and xz is α; the dihedral angle at the axis y, β; and at the axis z, γ: and if d be the cosine of the angle which a line perpendicular to the plane yz makes with x; e the cosine of the angle which a line perpendicular to xz makes with y; f the cosine of the angle which a line perpendicular to xy makes with z: and if θ be the angle of two planes whose equations are $Ax + By + Cz = m$, $A'x + B'y + C'z = m'$: we shall have (see Transactions of the Cambridge Philosophical Society, Vol. II. P. I. p. 200)

$$-\cos.\theta = \frac{\left\{\begin{array}{c}\dfrac{AA'}{d^2}+\dfrac{BB'}{e^2}+\dfrac{CC'}{f^2}\\ -\dfrac{A'B+AB'}{de}\cos.\gamma-\dfrac{A'C+AC'}{df}\cos.\beta-\dfrac{B'C+BC'}{ef}\cos.\alpha\end{array}\right\}}{\left\{\begin{array}{l}\left(\dfrac{A^2}{d^2}+\dfrac{B^2}{e^2}+\dfrac{C^2}{f^2}-\dfrac{2AB}{de}\cos.\gamma-\dfrac{2AC}{df}\cos.\beta-\dfrac{2BC}{ef}\cos.\alpha\right)\times\\ \times\left(\dfrac{A'^2}{d^2}+\dfrac{B'^2}{e^2}+\dfrac{C'^2}{f^2}-\dfrac{2A'B'}{de}\cos.\gamma-\dfrac{2A'C'}{df}\cos.\beta-\dfrac{2B'C'}{ef}\cos.\alpha.\right)\end{array}\right\}}$$

In the case of the rhomboid, since the dihedral angles are equal, α, β, γ are equal; and hence also d, e, f are equal. Hence

$$-\cos.\theta = \frac{AA'+BB'+CC'-(A'B+AB'+A'C+AC'+B'C+BC')\cos.\alpha}{\sqrt{\{(A^2+B^2+C^2-2(AB+AC+BC)\cos.\alpha)(A'^2+B'^2+C'^2-2(A'B'+A'C'+B'C')\cos.\alpha)\}}}$$

And if we put $p, q, r, p', q'; r'$ for A, B, C, A′, B′, C′, we shall have the angle.

If we have to find the angle of two planes resulting from the same law, $(p'; q'; r')$ will be a permutation of $(p; q; r)$; and the denominator of $-\cos.\theta$ will be

$$p^2+q^2+r^2-2(pq+pr+qr)\cos.\alpha.$$

We shall take examples of the use of these formulæ.

Ex. 1. To find the angle made by two planes of carbonate of lime resulting from the law* $(4, -5, -5)$. (*Chaux Carbonatée Cuboide* of Hauy).

The primary form of carbonate of lime is a rhomboid in which the angle α is $105^\circ 5'$, and therefore $\cos.\alpha = -.2602$.

Two of the secondary planes will be $(4; -5; -5)$ and $(-5; 4; -5)$, and if θ be the angle contained by these

$-\cos.\theta = \frac{-15-51\cos.\alpha}{66+30\cos.\alpha}$; or $\cos.\theta = \frac{5-17\times.2602}{22+10\times.2602} = .0297$

$\therefore \theta = 88^\circ.18.$

A variety of other rhomboids may be produced in this and other substances by other laws. In all cases, if two of the indices of the symbol be equal, as (p, q, q), there will only

* That this law is what Hauy calls a decrement on the inferior angles of 4 in breadth to 5 in height, and is in his notation represented by the symbol $e\frac{4}{5}$.

The angles obtained in the text differ slightly from these given by Hauy in consequence of his having assumed the angle of the primary rhomboid of carbonate of lime, $= 104^\circ.28'.40''$, for the convenience of using the cosine $= -\frac{1}{4}$.

be three coexistent planes; and if each of these planes be repeated, we shall have three pairs of parallel planes containing a rhomboid.

If the three indices in the symbol (p, q, r) be all different, we shall have six planes, and repeating each of these, we shall have a dodecahedron consisting of two six-sided pyramids. To this case belongs the following example:

Ex. 2. To find the angle of planes in carbonate of lime, resulting from the law $(1, -2, 0)$. (Decrement on the lateral edges by two rows in breadth. Symbol D^2. *Chaux Carbonatée Metastatique.* HAUY.)

Two adjacent* planes are $(1; -2; 0)$ $(1; 0; -2)$, and preserving the same notation as before

$$-\cos.\theta = \frac{1}{5 + 4\cos.\alpha} = -.2525, \theta = 104^\circ 38'.$$

By other laws we should find other dodecahedrons and their angles. But in many cases we have *two* laws, producing two sets of faces, and it may be required to find the angle between those of one set and of the other.

Ex. 3. To find the angles of planes $(2, -1, -1)$ and $(1, 0, 0)$. (Decrement by two rows in breadth on an inferior angle, combined with the primitive faces. Symbol e^2 P. *Chaux Carbonatée Imitable.* HAUY).

Adjacent faces* are $(2; -1; -1)$ and $1; 0; 0)$: and

$$-\cos.\theta = \frac{2 + 2\cos.\alpha}{\sqrt{(6 + 6\cos.\alpha)}} = 2\sqrt{\frac{1 + \cos.\alpha}{6}} = .7022; \theta = 134^\circ 37'.$$

9. We proceed now to the inverse problem; having given the angles of the secondary crystal to find the law of its planes. And we shall first suppose the secondary form to

* It will be shown afterwards how we may determine of co-existent planes which are adjacent.

be a rhomboid; in which case, as has already been observed, two of the indices in the symbol are equal.

PROP. Knowing the dihedral angles of the secondary rhomboid, to find the symbol of its planes,

Let (p, q, q) be the symbol of the planes, θ the angle of $(p\,;\,q\,;\,q)$ and $(q\,;\,p\,;\,q)$.

$$\therefore -\cos.\,\theta = \frac{2pq + q^2 - (p^2 + 2pq + 3q^2)\cos.\,\alpha}{p^2 + 2q^2 - 2(2pq + q^2)\cos.\,\alpha}$$

Here cos. θ being known, we have a quadratic equation to determine q in terms of p, which as the proportion $q : p$ only is wanted, is sufficient.

The equation will be

$$p^2(\cos.\,\theta - \cos.\,\alpha) + 2pq(1 - \cos.\,\alpha - 2\cos.\,\alpha\cos.\,\theta) + q^2(1 - 3\cos.\,\alpha + 2\cos.\,\theta - 2\cos.\,\alpha\cos.\,\theta) = 0$$

There will be for each value of θ two values of $\frac{q}{p}$, and therefore two laws according to which the same secondary form may be produced. It is to be noticed however, that the direction of the primitive faces, and consequently of the cleavage will be different in the two cases.

10. PROP. It is required to find according to what law we shall have a rhomboid similar to the primary one.

Here $\theta = \alpha$: therefore the first sum of the above equation vanishes, and the remaining part will be verified either by $q = 0$, or by

$$2p(1 - \cos.\,\alpha - 2\cos.^2\alpha) + q(1 - \cos.\,\alpha - 2\cos.^2\alpha) = 0, \text{ or } q = -2p.$$

Therefore $(1, 0, 0)$ and $(1, -2, -2)$ each give $\theta = \alpha$. The first indicates the primary face, and the form is the primary form. The other indicates a decrement by 2 in height on the inferior angle, which it appears gives a rhomboid identical with the primary rhomboid.

11 PROP Knowing the lateral angles made, at the terminal edges, by the planes of any bipyramidal dodecahedron to find the symbols.

If we have planes (p, q, r) they will generally form a bipyramidal dodecahedron, and the six angles at the edges of each pyramid will be alternately greater and less. If p, q, r be the order of magnitude of the indices, p being the greatest, the order of the faces will be that represented in fig. (see hereafter the section on the arrangement of faces). Hence faces occur in the order $(p; q; r)\ (q; p; r)\ (r; p; q)$ &c.: and if θ be the angle of the two first, and θ' of the next, we shall have

$$-\cos.\theta = \frac{2pq + r^2 - (p^2 + q^2 + 2pr + 2qr)\cos.\alpha}{p^2 + q^2 + r^2 - 2(pq + pr + qr)\cos.\alpha};$$

$$-\cos.\theta' = \frac{2qr + p^2 - (q^2 + r^2 + 2pq + 2pr)\cos.\alpha}{p^2 + q^2 + r^2 - 2(pq + pr + qr)\cos.\alpha}.$$

from which equations we have to determine q and r in terms of p.

To eliminate in these equations would lead to expressions of four dimensions, and it will generally be simpler to find q and r by trial. If we assume for p any number, as 12; q and r, which generally bear to it very simple ratios, will in most cases be whole numbers, and may be found by a few trials. And if the ratios of q and r to p involve quantities which are not divisors of 12, still the trials made on this supposition will indicate *nearly* the values of q and r; and by trying other values for p, we may obtain them accurately.

If two of the indices, as q, r be negative; the order of the faces will be $(p; -r; -q)(-r; p; -q)(-q; p; -r)$, &c. and the rest of the process will be the same as before.

12. PROP. Knowing the angles made by any plane with two primary planes, to find its symbol.

Let $(p; q; r)$ be the plane, and $(0, 1, 0)(0, 0, 1)$ the two primary planes; θ and θ' the given angles

$$\therefore \cos.\theta = \frac{q - (p+r)\cos.\alpha}{\sqrt{\left\{p^2+q^2+r^2-2(pq+pr+qr)\cos.\alpha\right\}}}$$

$$\cos.\theta' = \frac{r - (p+q)\cos.\alpha}{\sqrt{\left\{p^2+q^2+r^2-2(pq+pr+qr)\cos.\alpha\right\}}}$$

whence q and r must be found in terms of p, as in last proposition.

Or we may find them directly thus. Since one of the three p, q, r is indeterminate, assume $p^2+q^2+r^2-2(pq+pr+qr)\cos.\alpha = 1$.

$\therefore \cos.\theta = q - r\cos.\alpha - p\cos.\alpha$; $\cos.\theta' = r - q\cos.\alpha - p\cos.\alpha$.

Eliminating, we have

$$q\sin.^2\alpha = \cos.\theta + \cos.\alpha\cos.\theta' + p\cos.\alpha(1+\cos.\alpha);$$

$$r\sin.^2\alpha = \cos.\theta' + \cos.\alpha\cos.\theta + p\cos.\alpha(1+\cos.\alpha).$$

If we substitute these values in the assumed equation multiplied by $\sin.^4\alpha$, viz.

$$\left\{p^2+q^2+r^2-2(pq+pr+qr)\cos.\alpha\right\}\sin.^4\alpha = \sin.^4\alpha$$

we shall have a quadratic equation in p; and hence p, q, r are found.

13. Prop. To find what laws will give prisms parallel to the axis of the primary rhomboid.

For this purpose the planes must be parallel to the axis; and the equation of a plane must be consistent with the equations of the axis, which are

$$y = x,\ z = x.$$

Let $(p; q; r)$ be the plane; $\therefore px+qy+rz = 0$ is the equation to it, supposing it to pass through the origin; and since $y = x, z = x$; we have $px+qx+rx = 0$ $\therefore p = -(q+r)$.

If $r = q$, $p = -2q$; the planes are $(-2, 1, 1)$ and the

secondary rhomboid becomes a regular hexagonal prism. (Example. *Chaux Carbonatée Prismatique.* HAUY.)

In other cases the secondary form is an irregular hexagonal prism, the angles being equal, three and three alternately.

14. PROP. To find the symbol of a plane which truncates any edge of a given form.

Let two faces $(p\,;\,q\,;\,r)\,(p'\,;\,q'\,;\,r')$ meet, and let $(\mathrm{P}\,;\,\mathrm{Q}\,;\,\mathrm{R},)$ be a plane which truncates the edge formed by their intersection: the plane must be parallel to this intersection; and the equations to the intersection must be consistent with the equation $\mathrm{P}x + \mathrm{Q}y + \mathrm{R}z = 0$. Now for the intersection we have $px + qy + rz = 0$, $p'x + q'y + r'z = 0$; whence

$$(pq' - p'q)\,x = (qr' - q'r)z,\ (p'r - pr')x = (qr' - q'r)y.$$

Multiply $\mathrm{P}x + \mathrm{Q}y + \mathrm{R}z = 0$ by $(qr' - q'r)$ and substitute, and we have

$$\mathrm{P}(qr' - q'r) + \mathrm{Q}(p'r - pr') + \mathrm{R}(pq' - p'q) = 0.$$

And if P, Q, R fulfil this condition, $(\mathrm{P}\,;\,\mathrm{Q}\,;\,\mathrm{R})$ will be a plane truncating the edge as required.

15. PROP. To find the symbol of a plane which truncates an edge of any secondary rhomboid.

This is a particular case of last Prop. when instead of $(p\,;\,q\,;\,r)$ $(p'\,;\,q'\,;\,r')$, the planes are $(p\,;\,q\,;\,q)\,(q\,;\,p\,;\,q)$. Hence the equation of condition becomes

$$\mathrm{P}(q^2 - pq) + \mathrm{Q}(q^2 - pq) + \mathrm{R}(p^2 - q^2) = 0$$

or $$\mathrm{P}q + \mathrm{Q}q - \mathrm{R}(p + q) = 0$$

Hence if $\mathrm{R} = q$, $\mathrm{P} + \mathrm{Q} = p + q$, and with this condition, $(\mathrm{P}\,;\,\mathrm{Q}\,;\,q)$ is the plane required.

Ex. Required the planes which truncate the edges of the rhomboid produced by the law $(3, -1, -1)$.

Here $p + q = 2$; $\therefore$ the values which may be given to

P, Q are any number whose sum is 2. Thus (1, 1, — 1) (2, 0, — 1) are truncating faces.

(This rhomboid truncated by these two planes occurs in HAUY's *Chaux Carbonatée Progressive.* Fig. 41.)

The plane thus determined will always be parallel to the intersection of the two planes; but in order that it may truncate the edge, it must meet both of them on the really existing part of each plane. This condition is easily introduced in each particular case.

16. In order to express, by means of the symbols already introduced, any crystal whatever, we may write down the symbols of the faces by which it is bounded; indicating by the punctuation the permutations which are allowed. It will be convenient also to mark the number of the faces which arise from these permutations. In the rhomboid, when all the three indices are different, this number will be *six*. When two are alike, it it will be *three*. Thus (6) (p, q, r) may indicate that the crystal has six faces arising from the law expressed by (p, q, r) and (3) (p, p, r) may represent a crystal with three faces arising from the law (p, p, r); which is what would, according to HAUY, be called a decrement on an angle at the summit.

It often happens that faces in a crystal are repeated; that is, that there are faces parallel to one another, one of which may be considered as a repetition of the other. In that case we may distinguish them by placing a 2 before them as a multiplier. Thus 2 (3) (p, p, r) indicates a rhomboid produced by repeating each of the three faces represented by (p, p, r). This is in fact the mode in which a rhomboid is always produced. In the same manner 2 (6) (p, q, r) is the

symbol of a dodecahedron, which results from repeating each of the six planes (p, q, r).

§. 2. *The Quadrangular Prism.*

17. The quadrangular prism may be right or oblique, and its base may be a square, a rectangle, a rhombus, or a parallelogram. But in all cases we may take one of its angles, and make that the origin of co-ordinates; and taking two of our co-ordinates along two edges of the base, and the third along the length of the prism, we shall be able to express the secondary planes in the same manner as in the case of the rhomboid. There will however be some additional considerations to introduce, since the edges of the prism may be of different magnitudes; and its angles not being symmetrical like those of a rhomboid, we shall no longer have the same coexistent planes which we had in the former case.

In order to introduce the first consideration, let x and y, fig. 6, be the co-ordinates in the direction of the edges of the base, and z in that of the length of the prism. Let the space bounded by the co-ordinate planes be filled with small similar prisms, and let their edges in the directions x, y, z be a, b, c respectively. Let a secondary plane PQR be formed, by taking away h prisms along the edge x, k along y, and l along z; then the lengths of AP, AQ, AR will be ha, kb, lc respectively; and the equation to the plane will be

$$\frac{x}{ha} + \frac{y}{kb} + \frac{z}{lc} = 1.$$

If we call $\frac{1}{ha}$, A; $\frac{1}{kb}$, B; $\frac{1}{lc}$, C; we shall have the angle between any two planes by the formula, Art. 8; putting for α, β, γ and for d, e, f, their values. But if we make $\frac{1}{h}, -p$,

$\frac{1}{k} = q, \frac{1}{l} = r, (p\,;\,q\,;\,r)$ may still be taken for the symbol of the plane. In this case $\frac{p}{a}$, $\frac{q}{b}$, $\frac{r}{c}$, are the co-efficients of the equation to the plane, and are to be used for A, B, C in calculating the angles which the planes make with each other.

We shall use the following terms; a *rhombic prism* is one whose base is a rhombus: an *oblique rhombic prism*, fig. 8, is one in which the sides are not at right angles to the base, the angles of the sides, as BA z, CA z being equal. A *doubly oblique prism*, fig. 7, is one in which the angles of the sides at the base BA z, CA z are unequal. Prisms are called *square* or *rectangular* when their bases are so: and when the base is a parallelogram with unequal sides, and angles not right angles, the prism is called *oblique-angled*. Besides these we have a prism which we may call the *oblique rectangular prism*,* fig. 9, in which besides the two rectangular ends we have two sides, as $c\,z$ and the opposite one, also rectangles.

1. *The doubly-oblique Prism*, fig. 7.

18. In this, since the angles are all different, no one of the solid angles (A, B, C, D) is similar to another. Hence if a plane be formed on one of the angles, there is no plane necessarily formed on another angle; consequently a plane as $(p\,;\,q\,;\,r)$ or $(p\,;-q\,;-r)$ does not necessarily imply any *co-existent* plane, and the symbol is to be written with the mark (;) between the indices, to show that no permutations are allowed.

Let the edges of the subtractive prisms in last article be a,

* We might consider B z as the base of prism, by which means it would be a right oblique angled prism. But the method adopted in the text seems to be more natural and simple.

in the direction AB, b in the direction AC, c in the direction Az. Then putting $\frac{p}{a}$, $\frac{q}{b}$, $\frac{r}{c}$ for A, B, C in the formula, Art. 8, we shall have the angles made by secondary planes.

Conversely, knowing the angles made by secondary planes we may determine A, B, C, as before, and when we have found in crystals the same substance, various values of A, B, C, we have

$$\frac{q}{p} = \frac{\mathrm{B}\,b}{\mathrm{A}\,a}, \quad \frac{r}{p} = \frac{\mathrm{C}\,c}{\mathrm{A}\,a};$$

and a, b, c are to be assumed so that $q : p$ and $r : p$ may be numerical ratios as simple as possible.

2. *The oblique rhombic Prism*, fig. 8.

19. In this case the angles zAB, zAC, and the sides AB, AC are equal; and consequently the two faces zAB, zAC are symmetrical; and whatever secondary plane is formed with reference to one, we must have a co-existent plane corresponding to the other. Hence, if we have a plane $(p; q; r)$ we must have a plane $(q; p; r)$ and we may express both these by the symbol $(p, q; r)$ the (,) indicating that the co-ordinates x and y may be exhanged, z remaining the same. And this is true whether p, q, r be positive or negative.

Here having found p, q, and r we have $h\,a$, $k\,a$, $l\,c$, because a and b are equal, and their values are to be determined as before.

3. *The oblique rectangular Prism*, fig. 9.

20. Here the solid angles A and C are similar in all respects, A being contained by two right angles BAC, CAz and the angle BAz, and C by the angles DCA, ACo, oCD equal to them. Hence whatever plane be formed on A, we must have a coexistent plane on C, agreeing with it, except

that the ordinate in AC is in the opposite direction: that is $(p\,;\,q\,;\,r)$ $(p\,;-q\,;\,r)$ are co-existent planes. These may be included in the formula $(p\,;\pm q\,;\,r)$.

4. *The right oblique-angled Prism*, fig. 10.

21. It is obvious that the opposite angles A and D of the base of this prism are similar in all respects; and with any secondary plane formed on one of them, we must have a co-existent similar plane on the other. That is, we must have a second plane, when x and y are negative, as they were positive in the first. Hence $(p\,;\,q\,;\,r)$ $(-p\,;-q\,;\,r)$ are co-existent planes; and we may express them thus $(\pm p\,;\pm q, r)$ it being understood in such symbols that the upper signs are taken together, and the lower together.

5. *The right rhombic Prism*, fig. 10.

22. Here, the opposite angles A, D are similar, and also the adjacent sides. Hence with a plane $(p\,;\,q\,;\,r)$ we have co-existent planes $(-p\,;-q\,;\,r)$ $(q\,;\,p\,;\,r)$ $(-q\,;-p\,;\,r)$. These may be included in the symbol $(\pm p, \pm q\,;\,r)$ the upper signs being taken together as before, and p, q being permutable as is indicated by the comma.

6. *The right rectangular Prism*, fig. 11.

23. Here the four angles A, B, C, D are similar. Hence $(p\,;\,q\,;\,r)$ has co-existent planes

$$(-p\,;\,q\,;\,r)\ (p\,;-q\,;\,r)\ (-p\,;-q\,;\,r)$$

These may be included in the formula

$$\left(\underline{\underline{\pm}}\, p\,;\ \underline{\pm}\, q\,;\,r\right)$$

the signs being taken in horizontal pairs.

7. *The right square Prism.*

24. In this case, besides the co-existent planes which we have in the last figure, we shall have those which arise from considering that the sides AB, AC are symmetrical, that is p and q are permutable. Here the symbol is $\left(\begin{matrix}+\\+\\=\end{matrix} p, \begin{matrix}\pm\\\pm\end{matrix} q; r\right)$ this will give eight secondary faces.

8. *The Cube.*

25. This differs from the last in having the edge in the direction z similar to those in x and y. Hence p, q, r may be permuted and the symbol is $\left(\begin{matrix}+\\+\\=\end{matrix} p, \begin{matrix}\pm\\\pm\end{matrix} q, r\right)$ which gives 24 secondary faces.*

There is no necessity to vary the sign of r, for the plane $(p; q; -r)$ is the same as $(-p; -q; r)$.

§ 3. *The regular Tetrahedron and Octahedron.*

26. In this and other cases where the figure is bounded by more than three planes we shall make three of the primary faces co-ordinate planes, and the remaining primary faces will be expressed by different symbols. Also the co-existent planes will be differently represented accordingly as they are on one angle or another, and we shall in each case have to determine the different forms which will thus occur.

Let Axyz, fig. 12, be a regular tetrahedron, and let Ax, Ay, Az be three co-ordinates.

* In some cases however, we have only half the number of faces which the law of symmetry would give. Thus in the case of the pentagonal dodecahedron derived from the cube, the law is (2, 1, 0); but the faces which occur are (2; 1; 0) (1; 0; 2) (0; 2; 1) which by the changes of sign become 12. The other 12 which arise from the symbols (1; 2; 0) (2; 0; 1) (0; 1; 2) are excluded.

Let a plane $p\,q\,r$ be formed on the angle A; then, since all the angles are symmetrical, we must have a coexistent plane at any other angle, as x.

Let $Ap = h$, $Aq = k$, $Ar = l$; and let $x\,P = h$, $x\,Q = k$, $x\,R = l$; it is required to find the equation to the plane PQR.

Draw $x\,M$ and $y\,K$ parallel to PQ and we have, if $Ax = a$, $x\,K = xy \cdot \frac{x\,P}{x\,Q} = a\frac{h}{k}$; $\therefore\ AK = a\left(1 - \frac{h}{k}\right)$.

Also $AM = Ay \cdot \frac{Ax}{AK} = \frac{a}{1 - \frac{h}{k}}$

Similarly if $x\,N$ be parallel to PR, we shall find $AN = \frac{a}{1 - \frac{h}{l}}$

Hence the equation of the plane $N\,x\,M$ is

$$\frac{x}{a} + \left(1 - \frac{h}{k}\right)\frac{y}{a} + \left(1 - \frac{h}{l}\right)\frac{z}{a} = 1;$$

or
$$\frac{x}{h} + \left(\frac{1}{h} - \frac{1}{k}\right)y + \left(\frac{1}{h} - \frac{1}{l}\right)z = \frac{a}{h}.$$

And the symbol of this plane will be

$$\left(\tfrac{1}{h};\ \tfrac{1}{h} - \tfrac{1}{k};\ \tfrac{1}{h} - \tfrac{1}{l}\right).$$

And the plane PQR is parallel to $N\,x\,M$, and will have the same symbol.

If $\frac{1}{h} = p$, $\frac{1}{k} = q$, $\frac{1}{l} = r$, the symbol of the plane PQR will be $(p;\ p - q;\ p - r)$.

In the same way we shall have at the angles y and z, planes

$$(q - p;\ q;\ q - r) \text{ and } (p - r;\ q - r;\ r).$$

But the edges Ax, Ay, Az are also similar, and therefore p, q, r may be permuted in any manner. Hence we have these co-existent planes

$$(p, q, r), (p, p - q, p - r), (q - p, q, q - r), (r - p, r - q, r).$$

It being understood that in each parenthesis the indices which are separated by commas may undergo any permutation.

The first symbol (p, q, r) gives 6 planes, and the three others also 6 each, making in all 24.

If the primary form be known to be a regular tetrahedron, it is evident that the first symbol (p, q, r) must be understood as implying also the rest. But in order to express all the planes we may include them in one symbol thus

$$\{(p, q, r)(p, p-q, p-r) \&c.\}$$

the &c. implying the coexistent planes.

27. PROP. To determine the symbol of the planes which truncate the *edges* of a tetrahedron.

The plane truncating the edge x is $(0, q, r)$: and hence by last article the general symbol includes the planes

$$(0, q, r), (q, q, q-r), (r, r-q, r)$$

which gives 12 planes. We omit $(0, -q, -r)$, which is identical with $(0, q, r)$.

If $q = r$ the planes are expressed by $(0, q, q)$, which gives 3 planes; but in order to truncate the six edges, each is used twice, and the symbol is $2(3)(0, q, q)$.

The *regular octahedron* is bounded by the same 4 planes as the tetrahedron, each being used twice; and its symbol is $2(4)\{(1, 0, 0)(1, 1, 1)\}$.

Its edges are also parallel to the edges of the tetrahedron, each being used twice. And any plane which can be deduced from the octahedron, may with equal simplicity be deduced from the tetrahedron.

28. PROP. In the regular tetrahedron to find the angle contained by planes $(0, 1, 1)$.

The plane angles of the tetrahedron are 60°; and hence, to find its dihedral angles, we have to find the angle of an equilateral spherical triangle whose sides are 60°. If α be this angle, we have

$$\therefore \cos. \alpha = \text{cotan. } 60 \,.\, \tan. 30 = \tan.^2 30 = \frac{1}{3}.$$

Let θ be the angle of the planes (0, 1, 1) (1, 0, 1), and we have by the formula

$$-\cos. \theta = \frac{1 - 3 \cos. \alpha}{2 - 2 \cos. \alpha} = 0 \text{ because } \cos. = \frac{1}{3}.$$

Hence the angle of the planes is a right angle. And in the same manner the angles made by the other planes will be right angles. The figure will be a *cube* bounded by the 3 planes (0, 1, 1) twice repeated.

Irregular Tetrahedrons and Octahedrons.

29. If we have an octahedron composed of two right quadrilateral pyramids, similar and equal, set base to base, we shall call this a *right octahedron;* and it will be termed *square*, *rectangular*, or *rhombic*, when the base is so. The tetrahedron, from which the right rectangular octahedron is derived, may be called the *direct symmetrical tetrahedron;* and that from which the right rhombic octahedron is derived, may be called the *inverse symmetrical tetrahedron*, on account of properties which will be explained immediately. Also, all the planes which can be derived from the octahedrons, may be derived more simply from the corresponding tetrahedrons; and we shall find the coexistent planes, and the angles made by the faces, in the same manner as in the previous cases.

§ 4. *Direct symmetrical Tetrahedron and rectangular Octahedron.*

30. Let A x y z, fig. 13, be a tetrahedron, and let all its edges be bisected, and the bisections joined by lines drawn in the faces. We shall thus have an octahedron DEFGHK. If we consider EFHK as the common base of the two pyramids of which the octahedron is composed, when EFHK is a rectangle, the octahedron is called rectangular; and when EFHK is a square, the octahedron is called square.

Let EFHK be a rectangle, the octahedron being a right one. Then all the faces of the octahedron will be isosceles triangles, of which DEF, DHK, GFE, GHK will be equal to each other, and the other four also equal to each other. Also, it is easily seen that the triangle A y z has its sides double of those of EFG, and is similar to it; and similarly x y z has its sides double of KHG. Therefore the two triangles A y z, x y z are both isosceles, (y z being the base,) and are equal in every respect; and similarly y A x and z A x are isosceles triangles equal in every respect.

Hence the solid angles at y and z are equal in every respect, and also those at A and x. And a plane passing through A x and through the middle of y z would divide the tetrahedron symmetrically into two equal portions. Hence we have called this the direct symmetrical tetrahedron.

We may suppose the solid angle A to be filled with parallelepipeds, the planes of which are parallel to the planes A x y, A x z, A y z, in the same manner as the solid angle A, fig. 1. And by removing these parallelepipeds according to any law, as in fig. 1, we obtain a secondary plane, of which the symbol and the equation may be known from the law.

31. But since the solid angles at A and at x are symmetrical, for every plane at A we shall have a co-existent plane at x,* of which we shall find the equation.

We may as before suppose Ax, Ay, Az, to be co-ordinates, and with any plane pqr at A we shall have a co-existent plane PQR at x, such that xP, xQ, xR are equal to Ap, Aq, Ar respectively.

PROP. The symbol of pqr being $(p; q; r)$ to find the symbol of PQR.

Let the small component parallelepipeds have the edge in direction $Ax = a$, and the edges in directions Ay, Az each $= c$ (these being equal). Also, let $Ax = na$, $Ay = Az = nc$.† And let the plane pqr be obtained by taking away h molecules in the direction Ax, k in the direction Ay, and l in the direction Az. Therefore $Ap = ha$, $Aq = kc$, $Ar = lc$: and the equation to the plane pqr is

$$\frac{x}{ha} + \frac{y}{kc} + \frac{z}{lc} = 1;$$

* The parallelepipeds of which the solid is supposed to be made up at x, are not in the same position with those of which it is supposed to be made up at A. Those at x are bounded by planes parallel to Axy, Axz, yxz, as those at A are by the planes which meet at A. If the crystal be divisible according to all the planes of a tetrahedron or octahedron, there are four different kinds of parallelepiped of which it may be conceived to be composed, corresponding to the four angles A, x, y, z. And we may take any one of these kinds with equal propriety. In fact, the mode of conceiving secondary planes to be formed by removing parallelepipeds, is an assumption to be considered right only so far as it exhibits the dependence of secondary planes upon the simplicity of the ratios $p : q : r$.

† If we suppose $Axyz$ to be made up of parallelepipeds, Ax, Ay, and Az having *equal numbers* of them, planes parallel to xyz will pass through all their angles. And if instead of parallelepipeds, we suppose that we have only points in space where the angles of the parallelepipeds would be, the planes which are determined by any adjacent three points will be the four planes, Axy, Axz, Ayz, xyz.

or if $p = \frac{1}{h}, q = \frac{1}{k}, r = \frac{1}{l}$,

$$p\frac{x}{a} + q\frac{y}{c} + r\frac{l}{e} = 1,$$

the symbol of which is $(p\,;\,q\,;\,r)$.

Draw $y\,O$, $x\,M$ parallel to PQ, meeting $A\,x$ and $A\,y$. Then

$$x\,O = \frac{xy \cdot xP}{xQ} = \frac{nc \cdot ha}{kc} = \frac{nha}{k}$$

$$\therefore AO = A\,x - x\,O = na\left(1 - \frac{h}{k}\right)$$

$$A\,M = \frac{A\,x \cdot A\,y}{A\,O} = \frac{na \cdot nc}{na\left(1 - \frac{h}{k}\right)} = \frac{nkc}{k-h}$$

Similarly if $x\,N$ be parallel to PR, $AN = \frac{nlc}{l-h}$.

Hence the equation to the plane $x\,M\,N$ is

$$\frac{x}{na} + \left(1 - \frac{k}{k}\right)\frac{y}{nc} + \left(1 - \frac{h}{l}\right)\frac{z}{nc} = 1$$

$$\text{or } p\frac{x}{a} + (p-q)\frac{y}{c} + (p-r)\frac{z}{c} = pn$$

and the equations to planes $p\,q\,r$ and PQR are

$$p\frac{x}{a} + q\frac{y}{c} + q\frac{z}{c} = 1\,;$$

$$p\frac{x}{a} + (p-q)\frac{y}{c} + (p-r)\frac{z}{c} = m$$

and their symbols are $(p\,;\,q\,;\,r)$, $(p\,;\,p-q\,;\,p-r)$.

Also the edges $A\,y$, $A\,z$ are symmetrical; and hence we have two other co-existent planes $(p\,;\,r\,;\,q)\,(p-r\,;\,p-q)$.

These are included in the formula $\left\{(p\,;\,q,r)(p\,;\,p-q,p-r)\right\}$

The solid angles at y and z are also symmetrical; and a plane being supposed to be formed at y as before, we must have a co-existent plane at z. Let $p'\,q'\,r'$ be a plane cutting off the angle y, and b being the edge of a molecule in the direction $y\,z$, let $y\,p', y\,q', y\,r' = hb, kc, lc$ respectively, and let $z\,P', z\,Q', z\,R' = y\,p', y\,q', y\,r'$ respectively. Then $p'\,q'\,r'$

and P′ Q′ R′ will be co-existent planes; and the condition of their co-existence is included in the preceding symbol.

The quantities a, b, c are as na, nb, nc, that is as Ax, yz and Ay. Or, referring to the octahedron in fig. 13, they are as FH, FE, and FD.

The square Octahedron.

32. When EFHK, fig. 13, is a square, Ax, yz will be equal, and the solid angles at y and z will be symmetrical to those at A and x, and will be similarly affected. Hence for a plane at A there will be co-existent planes at y and z.

Prop. To find the symbols of co-existent planes in this case,

If we take zP′, zQ′, zR′, $= yp', yq', yr', = Ap, Aq, Ar$ respectively, we shall, as in last article, find the equation of the planes $p'q'r'$, P′ Q′ R′ to be

$$\left(1-\frac{k}{l}\right)\frac{x}{na}+\frac{y}{nc}+\left(1-\frac{k}{h}\right)\frac{z}{nc}=m$$

$$\left(1-\frac{k}{l}\right)\frac{n}{na}+\left(1-\frac{k}{h}\right)\frac{y}{nc}+\frac{z}{nc}=m'$$

and since $p=\frac{1}{h}$, $q=\frac{1}{k}$, $r=\frac{1}{l}$, these are equivalent to

$$(q-r)\frac{x}{a}+q\frac{y}{c}+(q-p)\frac{z}{c}=\frac{mn}{k}$$

$$(q-r)\frac{x}{a}+(q-p)\frac{y}{c}+q\frac{z}{c}=\frac{m'n}{k}$$

Hence with a plane $(p\,;q\,;r)$ we have co-existent planes

$(q-r\,;q\,;q-p)$ and $(q-r\,;q-p\,;q)$.

But we have also a co-existent plane $(p\,;r\,;q)$ and therefore also $(r-q\,;r\,;r-p)$ and $(r-q\,;r-p\,;r)$

Hence in the square octahedron we have co-existent planes which may be included in this symbol

$$\{(p\,;q,r)\,(p\,;p-r,p-q)\,(q-r\,;q,q-p)\,(r-q\,;r,r-p)\}$$

All which are implied in $(p\,;q\,;r)$.

33. PROP. Having given the symbol of a plane derived from the tetrahedron, to find the manner in which it cuts the octahedron, Fig. 13.

Let PQR be any plane at the angle A; and let PQ meet DK and DE in S and T. $\therefore DS = \frac{DP \cdot AQ}{AP} = DP \cdot \frac{kc}{ha} = DP \cdot \frac{p}{q} \cdot \frac{c}{a}$.

And drawing QL parallel to DE, $DF = \frac{QL \cdot DP}{PL}$

Also $QL = AQ$ and $PL = AP - AL = AP - AQ \cdot \frac{Ax}{Ay} = ha - kc \cdot \frac{a}{c} = (h-k)a$

$$\therefore DT = DP \cdot \frac{AQ}{PL} = DP \cdot \frac{kc}{(h-k)a} = DP \cdot \frac{p}{q-p} \cdot \frac{c}{a}.$$

In the same way we find the portions cut off from DH and DF: and hence it appears that a plane $(p; q; r)$ cuts off from the four edges, which meet at the vertex D of the pyramid, lines which, parallel to the edges in the directions Ay, Az, xy, xz, are as

$$\frac{1}{q}, \frac{1}{r}, \frac{1}{q-p}, \frac{1}{r-p}$$

In whatever manner the plane DEF is cut by the plane PQR, the plane DHK will be similarly cut by the co-existent plane at x.

34. Hence, knowing the law by which a secondary face is derived from the octahedron, we can find its symbol.

The primary form is a square octahedron; to find the symbol of the face 'E' (*Ex. Zircon unibinaire*, HAUY).

This plane is drawn cutting off the angle E, in such a manner that the portions cut from EF, EG are double of those from EK, ED respectively; and the section on the face EFG parallel to FG or to Ay,

Since the part cut from EG, parallel to Az, is double of that

from ED, parallel to xy, and is in the negative direction, $\frac{1}{r} = -\frac{2}{q-p}$ or $p - q = 2r$.

Also since the section is parallel to Ay we must have $q = 0$.

Hence (2 ; 0 ; 1) is the symbol required. And the co-existent planes are

$$(2\ ;\ 0,\ 1)\ (2\ ;\ 1,\ 2)\ (-1\ ;\ 0, -2)\ 1\ ;\ -1,\ 1)$$

each of the parentheses gives two planes, and hence we have 8 arising from this law.

35. To find the angles which these planes make with the planes of the octahedron.

Example. *Zircon unibinaire*, HAUY.

In the square octahedron, which has been considered as the primary form of zircon, the angle of two adjacent faces of a pyramid is 123° 15′, and the angle of two opposite faces measured over the summit is 95° 40′. (PHILLIPS).

Hence the dihedral angle at Ax, which is (α) the angle of the planes EFK, FDH, is 95° 40′. And (β,) the angle at Ay is the angle of DEK, FEG, and is therefore the supplement of the angle of HFG, EFG, and it is therefore = 56° 45′. In the same manner (γ) the dihedral angle at Az is 56° 45′.

In order to apply the formulæ of Art. 8, we must find the values of d, e, f. Let XYZ, fig. 15, be a spherical triangle made by describing a sphere with center A, meeting Ax, Ay, Az in X, Y, Z. Then if XD be drawn perpendicular to YZ, d = sin. XD, similarly if YE be perpendicular on ZX, e = sin. YE, and $f = e$.

Now by NAPIER's rules, since XYD = 56° 45′, and YXD = $\frac{1}{2}$(95° 40′) = 47° 50′, r . cos. 56° 45′ = cos. XD . sin. 47° 50

$\therefore d$ = sin. 42° 11 ; d = .6730125.

Also $r.\cos.XY = \text{cotan.}\ 56°\,45'.\text{cotan.}\ 47°\,50'$ $\therefore xy = 43°\,37'$
and $YXE = 180 - 90°\,40' = 84°\,20'$
$\therefore r.\sin.YE = \sin.XY.\sin.84°20'$ $\therefore e = \sin.43°\,21'$; $e = .6864532$

The two planes of which we have to find the angle, are $(2\,;\,0\,;\,1)\,(1\,;\,0\,;\,0)$.

Hence by the formula, Art. 8,

$$-\cos.\theta = \frac{\frac{2}{d} - \frac{\cos.\beta}{f}}{\sqrt{\left\{\frac{4}{d^2} + \frac{1}{f^2} - \frac{4\cos.\beta}{df}\right\}}} = \frac{2f - d\cos.\beta}{\sqrt{\left\{4f^2 - 4fd\cos.\beta + d^2\right\}}}$$

To find θ, let $\tan.\omega = \frac{2f - d\cos.\beta}{d\sin.\beta} = \frac{2f}{d\sin.\beta} - \text{cotan.}\ \beta$; and we shall have, $-\cos.\theta = \frac{\tan.\omega}{\text{sec.}\ \omega} = \sin.\omega$. $\therefore \theta = 90° + \omega$.

By the values above given, we shall find $\omega = 60°\,48'$ and $\therefore \theta = 150°\,48'$. The value given by Mr. PHILLIPS is $150°\,12'$.

It may be observed, that $(2\,;\,0\,;\,1)$ is the side adjacent to the primary plane $(1\,;\,0\,;\,0)$; and that we obtain sides adjacent to other faces by taking *corresponding* co-existent planes from the formulæ in Art. 32.

Thus the primary faces $(1\,;\,0\,;\,0)$ have adjacent secondary faces $(2\,;\,0\,;\,1)$ and $(2\,;\,1\,;\,0)$.
The primary faces $(0\,;\,1\,;\,0)$ have adjacent $(1\,;\,2\,;\,0)$ and $(1\,;\,-1\,;\,1)$
The primary faces $(0\,;\,0\,;\,1)$ have adjacent $(1\,;\,0\,;\,2)$ and $(1\,;\,1\,;\,-1)$
The primary faces $(1\,;\,1\,;\,1)$ have adjacent $(2\,;\,1\,;\,2)$ and $(2;\,2;\,1)$

Here instead of $(-1\,;\,0\,;\,-2)$ &c. we have written $(1\,;\,0\,;\,2)$ &c. which represents the same plane.

§ 5. *Inverse symmetrical Tetrahedron and rhombic Octahedron.*

36. Let A $x\,y\,z$, fig. 16, be a tetrahedron; and let its edges be bisected, and an octahedron formed as before. In this octahedron, let EFHK be the rhombic base; and the two

pyramids which compose the octahedron being right ones and equal, it is evident that the four lines DE, EG, GH, HD will be equal, and the four lines DF, FG, GK, KD. Now Ax is double of FH, and xy of HK. Hence A$x = yz$. Similarly A$y = xz$, and A$z = xy$. Hence it appears that the four triangles which form the sides of the tetrahedron have their sides equal respectively, and are therefore equal and similar. Hence the four solid angles A, x, y, z, are contained by equal angles, and are symmetrical. Thus the angles xAy, yAz, zAx are equal to Axz, yxz, Axy. And this tetrahedron may be called an inverse symmetrical tetrahedron.

From the law of symmetry, whatever plane is formed at the angle A, we must have a coexistent plane at each of the angles x, y, z, the equal and opposite edges being similarly affected.

37. Prop. A plane $(p;q;r)$ being known, to find the coexistent planes. Fig. 17.

Let Ax, Ay, Az be na, nb, nc.

Ap, Aq, A$r = ha$, kb, lc; and $p = \frac{1}{h}, q = \frac{1}{k}, l = \frac{1}{r}$.

$\therefore x$P, xQ, xR are ha, kb, lc.

Draw yO, xM parallel to PR.

$$xO = xy \cdot \frac{xP}{xR} = nc\,\frac{ha}{lc} = \frac{nha}{l}; \quad AO = na\left(1 - \frac{h}{l}\right)$$

$$AM = Ax \cdot \frac{Ay}{AO} = \frac{nb}{1 - \frac{h}{l}} = \frac{nb}{1 - \frac{r}{p}}$$

Similarly if xN be parallel to PQ, $AN = \frac{nc}{1 - \frac{h}{k}} = \frac{nc}{1 - \frac{q}{p}}$.

Hence the equation of the plane xNM, which is parallel to PQR, is

$\frac{x}{na}+\left(1-\frac{r}{p}\right)\frac{y}{nb}+\left(1-\frac{q}{p}\right)\frac{z}{nc}=1$ or $p\frac{x}{a}+(p-r)\frac{y}{b}+(p-q)\frac{z}{c}=np$

and its symbol is $(p\,;p-r\,;p-q)$.

In the same manner the angle y gives a plane $(q-r\,;q\,;q-p)$ and the angle z a plane $(r-q\,;r-p\,;r)$.

Hence the co-existent planes are

$(p\,;q\,;r),(p\,;p-r\,;p-q),(q-r\,;q\,;q-p),(r-q\,;r-p\,;r)$.

These four planes would truncate symmetrically the four faces of one of the pyramids which compose the octahedron, and planes parallel to them would truncate similarly the planes of the other pyramid.

38. PROP. To find the portions cut from the edges of the octahedron by the plane $(p\,;q\,;r)$.

Let the plane P, Q, R, fig. 16 and 18, meet DK, DE, DF, DH in S, T, U, V. Draw QL parallel to DE. Then

$$DS=DP\cdot\frac{AQ}{AP}=DP\cdot\frac{kb}{ka}=DP\cdot\frac{p}{q}\frac{b}{a}$$

$$QL=AQ\frac{xy}{Ay}=kb\cdot\frac{c}{b}=kc,\ AL=AQ\frac{Ax}{Ay}=kb\cdot\frac{a}{b}=ka\,;\ PL=(h-k)a$$

$$DT=DP\cdot\frac{QL}{PL}=DP\frac{kc}{(h-k)a}=DP\cdot\frac{p}{q-p}\cdot\frac{c}{a}$$

Similarly DV and DU would be $DP\cdot\frac{p}{r}\cdot\frac{c}{a}$ and $DP\cdot\frac{p}{r-p}\cdot\frac{b}{a}$

Hence DS, DT, DU, DV are as $\frac{1}{q}\cdot b,\frac{1}{q-p}c,\frac{1}{r-p}b,\frac{1}{r}c$.

Hence for the four co-existent planes the edges cut off are respectively as

$$\frac{b}{q},\frac{c}{q-p},\frac{b}{r-q},\frac{c}{r}\,;$$

$$\frac{b}{r-p},\frac{c}{r},\frac{b}{q},\frac{c}{q-p}\,;$$

$$\frac{b}{q},\frac{c}{r},\frac{b}{r-p},\frac{c}{q-p}\,;$$

$$\frac{b}{r-p},\frac{c}{q-p},\frac{b}{q},\frac{c}{r}.$$

The calculations would be nearly the same as in the case of the square octahedron, article 35. We should have to calculate d, e, f from the angles of the octahedron. Thus in sulphur, according to Mr. PHILLIPS (p. 361) we have incidence of

GEF on GEK $= 106° 30'$; $\therefore$ angle at A $x = 73° 30' = \gamma$

GFH on GFE $= 85° 5'$; $\therefore$ angle at A $y = 94° 55' = \beta$

GHF on DHF $= 143° 25'$; $\therefore$ angle at A $z = 36° 35' = \alpha$

And if we construct a triangle, of which the three angles are α, β, γ, and draw arcs from these angles perpendicular on the opposite sides, the sines of these arcs will be respectively d, e, f. And by first finding the sides of the triangle by spherical trigonometry, these may be calculated.

§ 6. *The regular triangular Prism.* Fig. 19.

39. This is a right prism, having for its base an equilateral triangle. It includes the regular hexagonal prism by repeating the lateral faces.

PROP. To find the co-existent planes.

By the law of symmetry, for every plane on one angle A, we must have co-existent planes on x, y. Let $p\,q\,r$ be any plane whose symbol is $(p; q; r)$, and the lines A $p = h$, A $q = k$, A $r = l$, when $p = \frac{1}{h}$, $q = \frac{1}{k}$, $r = \frac{1}{l}$. Then we shall have a plane PQR where $x\,\mathrm{P} = h$, $x\,\mathrm{Q} = k$, $x\,\mathrm{R} = l$. Draw x M, y O parallel to PQ.

$$\therefore x\,\mathrm{O} = xy \cdot \frac{x\,\mathrm{P}}{x\,\mathrm{Q}} = \frac{h}{k} \text{ if } \mathrm{A}\,x = xy = \mathrm{A}\,y = 1.$$

$$\mathrm{AM} = \mathrm{A}\,x \cdot \frac{\mathrm{A}\,y}{\mathrm{AO}} = \frac{1}{1 - \frac{h}{k}} = \frac{1}{1 - \frac{q}{p}} = \frac{p}{p - q}.$$

Similarly if x N be parallel to RP, $\mathrm{AN} = \mathrm{A}\,x \cdot \frac{x\,\mathrm{R}}{x\,\mathrm{P}} = \frac{l}{h} = \frac{p}{r}$. Hence the equation of the plane x MN is

$x + \frac{p-q}{p} y - \frac{r}{p} z = 1$ or $px + (p-q)y - rz = p$.

$\therefore$ its symbol, or that of PQR, is $(p\,;\,p-q\,;\,-r)$.

Similarly, at y, we shall have a plane $(q-p\,;\,q\,;\,-r)$.

Also, since the edges Ax and Ay are symmetrical, we have a plane $(q\,;\,p\,;\,r)$. And hence the co-existent planes are $(p\,;q\,;r)\,(p\,;p-q\,;-r)(q-p\,;q\,;-r)(q\,;p\,;r)\,(q\,;q-p\,;-r)$ $(p-q\,;\,p\,;-r)$. Which may be included in the symbol

$$\left\{(p,q\,;\,r)\,(p,p-q\,;-r)\,(q,q-p\,;-r)\right\}$$

§ 7. *The rhombic Dodecahedron.*

40. If we take a regular tetrahedron $wxyz$, fig. 20, and from its centre of gravity A draw lines Aw, Ax, Ay, Az, the angles made by any two of these lines will be the same. And by taking planes passing through any two of these lines we shall have six planes symmetrically disposed, each of which will make an angle of 120° with four others. A figure bounded by planes parallel to these planes, each taken twice, and symmetrically disposed, will be the rhombic dodecahedron.

We may consider the three lines Ax, Ay, Az as axes of co-ordinates; and any plane pqr which cuts them must have co-existent planes cutting any two of them and Aw. Also, as the lines Ax, Ay, Az are similar, in a plane (p,q,r) we may present the indices in any manner.

41. PROP. To find the symbols of co-existent planes in the rhombic dodecahedron.

Let a plane pqr cut wA produced in O. Let x, y, z be the co-ordinates of the point O. The equations of the line Aw are $y=x, z=x$. And if the equation to the plane

$p\,q\,r$ be $p\,x + q\,y + r\,z = m$, we shall have the co-ordinates of the point O by combining these equations. Hence we have $p\,x + q\,x + r\,x = m$, or $x = \frac{m}{p+q+r}$.

But if the co-ordinates x, y, z be projected upon AO, we shall have $\mathrm{AO} = \mathrm{A}\,x \cos. x\,\mathrm{AO} + \mathrm{A}\,y \cos. y\,\mathrm{AO} + \mathrm{A}\,z \cos. z\,\mathrm{AO}$. And since $\cos. x\,\mathrm{AO} = \cos. y\,\mathrm{AO} = \cos. z\,\mathrm{AO} = \frac{1}{3}$, $\mathrm{AO} = \frac{x+y+z}{3} = x$. $\therefore \mathrm{AO} = \frac{m}{p+q+r}$.

Now let $p'x + q'y + r'z = m$ be the equation to a plane which cuts $\mathrm{A}\,x$, $\mathrm{A}\,y$, $\mathrm{A}\,w$ in the same manner in which $(p\,;q\,;r)$ cuts $\mathrm{A}\,x$, $\mathrm{A}\,y$, $\mathrm{A}\,z$. Therefore the portion cut off from $r\,y$ A produced will be $\frac{m}{p'+q'+r'}$. Also the portions from $\mathrm{A}\,x$ and $\mathrm{A}\,y$, are $\frac{m}{p'}$, $\frac{m}{q'}$.

Hence $\frac{m}{p'} = \frac{m}{p}$, $\frac{m}{q'} = \frac{m}{q}$, $-\frac{m}{p'+q'+r'} = \frac{m}{r}$

$\therefore p' = p$, $q' = q$, $p' + q' + r' = -r$. ; $r' = -\overline{p+q+r}$.

Hence if $(p\,;q\,;r)$ be a plane $(p\,;q\,;-\overline{p+q+r})$ is a co-existent plane.

Also the axes of x, y, z being symmetrical, $(p\,;q\,;r)$ has co-existent planes (p, q, r). And making $-\overline{p+q+r} = s$, we have the planes

$$(p, q, r)\,(p, q, s)\,(p, r, s)\,(q, r, s).$$

Each of these symbols gives six permutations, so that we have in all 24 co-existent planes.

§ 8. *On the arrangement of secondary faces.*

42. When crystals have faces determined by the laws considered in the preceding pages, they will have the form of polyhedrons bounded by polygons; and in order to determine the dihedral angles, &c. it will be necessary to know

in what order the faces occur, and which are adjacent. This may be done in the following manner:

Let AI fig. 21, be any parallelepiped of which the edges Ax, Ay, Az are a, b, c. Let an ellipsoid be described, of which the center is I, touching three planes of this parallelepiped in D, E, F. If we suppose any secondary plane, deduced from this parallelepiped, to be drawn so as to touch the ellipsoid in P, the situation of the points P will determine the position of the planes. Let $Ax + By + Cz = m$ be the equation to the plane. The equation to the ellipsoid will be $\frac{(a-x)^2}{a^2} + \frac{(b-y)^2}{b^2} + \frac{(c-z)^2}{c^2} = 1$.

And that the plane may touch the ellipsoid, the differential co-efficients $\left(\frac{dy}{dx}\right)$ and $\left(\frac{dz}{dx}\right)$ must be the same in both. Hence

$$\left(\frac{dy}{dx}\right) = -\frac{A}{B} = -\frac{b^2}{a^2}\cdot\frac{(a-x)}{(b-y)}; \quad \left(\frac{dz}{dx}\right) = -\frac{A}{C} = -\frac{c^2}{a^2}\cdot\frac{(a-x)}{(c-z)}.$$

Therefore $\frac{a-x}{A\,a^2} = \frac{b-y}{B\,b^2}$; $\frac{a-x}{A\,a^2} = \frac{c-z}{C\,c^2}$.

And substituting in the equation to the ellipsoid we have

$$\frac{1}{a^2}(a-x)^2 + \frac{B^2 b^2}{A^2 a^4}(a-x)^2 + \frac{C^2 c^2}{A^2 a^4}(a-x)^2 = 1$$

$$\therefore a - x = \frac{A\,a}{\sqrt{(A^2 a^2 + B^2 b^2 + C^2 c^2)}}$$

$$\therefore b - y = \frac{B\,b}{\sqrt{(A^2 a^2 + B^2 b^2 + C^2 c^2)}}$$

$$\text{and } c - z = \frac{C\,c}{\sqrt{(A^2 a^2 + B^2 b^2 + C^2 c^2)}}$$

Knowing the position of the points P for all the planes, we have the polyhedron, on the supposition that it is made such that the ellipsoid can be inscribed in it; which is always possible by supposing the planes to move parallel to themselves till they touch it.

We shall see more clearly the position of the points P if

we suppose it to be determined by angular distances like the longitude and latitude on a globe, assuming as the axis of the ellipsoid that about which the figure is symmetrical.

43. (1) In the rhomboid. Here $Ax = Ay = Az = 1$, suppose IA be taken as the axis; and a plane API being drawn, let the angle between this plane and IA x be called the longitude (λ) of the point P; and let the complement of AIP be called the latitude (μ) of P.

Let the co-ordinates of P be called X, Y, Z. Then the plane API has a point A, of which the co-ordinates are 0, 0, 0; a point I, of which the co-ordinates are 1, 1, 1; a point P, of which the co-ordinates are X, Y, Z. Hence its equation is $(Y - Z)x + (Z - X)y + (X - Y)z = 0$. And the equation to IA x is $y - z = 0$. Therefore by the formula for the angle of two planes, Art. 8,

$$-\cos.\lambda = \frac{-2X + Y + Z - (2X - Y - Z)\cos.\alpha}{\sqrt{\left\{2\left((Y-Z)^2 + (Z-X)^2 + (X-Y)^2 + 2(X^2 + Y^2 + Z^2 - XY - XZ - YZ)\cos.\alpha\right)\right\}}}$$

If the symbol of the plane be $(p; q; r)$ its equation is $px + qy + rz = m$; and hence

$a - X = \frac{pa}{\sqrt{(p^2 + q^2 + r^2)}}$; and similarly for Y and Z. Hence

$$\cos.\lambda = \frac{(2p - q - r)(1 + \cos.\alpha)}{2\sqrt{\left\{(p^2 + q^2 + r^2 - pq - pr - qr)(1 + \cos.\alpha)\right\}}};$$

$$= \frac{2p - q - r}{2\sqrt{(p^2 + q^2 + r^2 - pq - pr - qr)}} \cdot \sqrt{1 + \cos.\alpha}).$$

To find μ; if we draw PM perpendicular in AI, and call IP, r, we shall have $IM = r \sin.\mu$, and μ will be greater as IM is greater. Now if IM, NO, OP to the co-ordinates of P measured from I, and if we draw perpendiculars from N and O on IA, we shall see that $IM = (a - X)\cos.\zeta + (a - Y)\cos.\zeta + (a - Z)\cos.\zeta$ where ζ is the angle which AI makes with A x, A y or A z.

$$r \sin. \mu = \frac{p+q+r}{\sqrt{(p^2+q^2+r^2)}} \cdot \cos. \zeta.$$

By these formulæ we may determine the arrangement of any set or sets of secondary faces. Thus if we have a symbol (p, q, r) in which $p > q$, $q > r$; we have 6 faces. The expression for r sin. μ is the same for all: hence they are all at the same distance from the summit B. And cos. λ will be greater as $2p - q - r$ is, or as $3p - (p+q+r)$ is so. Consequently the values of cos. λ taken in order of magnitude will correspond to $(p; q; r)\,(q; p; r)\,(r; p; q)$. The other three values be the same, viz. $(p; r; q)\,(q; r; p)\,(r; q; p)$; and indicate longitudes on the other side of A x.

The arrangement of the planes is represented in fig. 22.

It is to be observed that as the order of the *first* index is p, q, r, beginning from x, the order of the *second* index is p, q, r beginning from y, and of the third p, q, r, beginning from z.

44. (2) In the Prism. Let the line IF, fig. 21, parallel to A z, be taken for the axis of the ellipsoid; and let the position of P be determined by (λ) the longitude which is measured by the angle between the planes FID and FIP; and by μ the latitude, the angle PIN.

It is evident that tan. λ will be greater as $\frac{NO}{IO}$ is greater. Let $(p; q; r)$ be the symbol of the plane, and its equation will be $\frac{px}{a} + \frac{qy}{b} + \frac{rz}{c} = 1$.

And the values of IO, ON, NP, will be

$$\frac{pa}{\sqrt{(p^2+q^2+r^2)}}, \frac{qb}{\sqrt{(p^2+q^2+r^2)}}, \frac{rc}{\sqrt{(p^2+q^2+r^2)}}.$$

Hence tan. λ will be greater as $\frac{qb}{pa}$ is greater; or as $\frac{q}{p}$ is greater; because a and b are constant for the same substance. Also sin. μ is greater as PN is greater; that is, as $\frac{rc}{\sqrt{(p^2+q^2+r^2)}}$ is so.

And hence we may arrange the faces in the order of their longitude and latitude.

We might in the same manner find the position of the planes for other primitive forms, but what has been done will generally be sufficient.

§ 9. *On the angles made by edges.*

45. If we have two lines referred to any co-ordinates, of which the equations are $y = Ax$, $z = Bx$; $y = A'x$, $z = B'x$; and if the plane angles of the faces be known; viz. the angle which x makes with $y = \varphi$, the angle which x makes with $z = \psi$ and the angle which y makes with $z = \omega$; we shall find θ, the angle which the two lines make with one another, by the formula,

$$\cos.\theta = \frac{1 + AA' + BB' + (A + A')\cos.\varphi + (B + B')\cos.\psi + (A'B + AB')\cos.\omega}{\sqrt{(1 + A^2 + B^2 + 2A\cos.\varphi + 2B\cos.\psi + 2AB\cos.\omega)(1 + A'^2 + B'^2 + 2A'\cos.\varphi + 2B'\cos.\psi + 2[A'B'\cos.\omega)}}$$

(See Trans. of Camb. Phil. Soc. vol. ii; P. I; p. 202.)

When we know the symbols of the planes, the co-efficients A, B will be found by eliminating y and z in the equations of the planes where intersection is considered.

Ex. In a rhomboid it is required to find the angles made by the opposite edges of a pyramid formed of planes (p, q, r).

By referring to fig. 22. it will be seen that opposite edges are those which are produced by intersections of planes $(p\,;\,q\,;\,r)(q\,;\,p\,;\,r)$ and $(q;\,r;\,p)(r\,;\,q\,;\,p)$.

To find the equation to the first line we have

$$px + qy + rz = 0$$
$$qx + py + rz = 0$$

whence $y = x$, $z = -\frac{p+q}{r}x$.

In the same manner we should find for the second line

$$y = x,\ z = -\frac{q+r}{p}x.$$

Substituting for A, B, A′, B′ in the formula, we have, since $\varphi = \psi = \omega$

$$\cos.\theta = \frac{1+1+\frac{(p+q)(q+r)}{pr}+2\cos.\varphi-\left(\frac{p+q}{r}+\frac{q+r}{p}\right)\cos.\varphi-\left(\frac{p+q}{r}+\frac{q+r}{p}\right)\cos.\varphi}{\sqrt{\left(1+1+\frac{(p+q)^2}{r^2}+2\cos.\varphi-2\frac{p+q}{r}\cos.\varphi-2\frac{p+q}{r}\cos.\varphi\right)\left(1+1+\frac{(q+r)^2}{p^2}+2\cos.\varphi-2\frac{q+r}{p}\cos.\varphi-2\frac{q+r}{p}\cos.\varphi\right)}}$$

$$= \frac{3pr+pq+q^2+qr-2(p^2+r^2+pq+qr-pr)\cos.\varphi}{\sqrt{((p+q)^2+2r^2-2(p+q-r)r\cos.\varphi))\,((q+r)^2+2p^2-2(q+r-p)p\cos.\varphi)}}$$

And if we take any other opposite pairs of planes, $(p;r;q)$ $(q;r;p)$ and $(q;p;r)(r;p;q)$; or $(r;q;p)(r;p;q)$ and $(p;r;q)$ $(p;q;r)$; we shall have the same value for θ. Hence this angle may be used as the characteristic of a pyramid produced by any such law from a rhomboid: and consequently of a dodecahedron resulting from repeating the faces of the pyramid. It is employed in this manner by BOURNON in characterising the dodecahedrons of carbonate of lime.

§ 10. *Recapitulation.*

46. It may be useful to collect in one view the results of the foregoing investigations. If we take a solid angle of the primary form of a crystal for the origin, and the three edges for three co-ordinates, any secondary plane may be obtained by removing a pyramid, the edges of which consist of h, k, l, molecules respectively. If we make $p = \frac{1}{h}$, $q = \frac{1}{k}$, $r = \frac{1}{l}$, the secondary plane may be represented by $(p;q;r)$ which will express its position without determining its distance from the origin: p, q, r may be positive, 0, or negative. By the law of symmetry with respect to the angles and edges of primary forms, if one secondary plane exist, certain others must also exist, which are hence called *co-existent* planes. Some of these are obtained by permuting the order of the letters in the symbol (p, q, r); and the instances where this

permutation is allowed may be distinguished from those where it is not, by separating the letters p, q, r in the former case by a comma, and in the latter by a semicolon. The other co-existent planes in each primary form will be seen in the following table.

Table of planes which exist if $(p\,;\,q\,;\,r)$ exist.

In the rhomboid	(p, q, r)
The doubly-oblique prism	$(p\,;\,q\,;\,r)$
The oblique rhombic prism	$(p, q\,;\,r)$
The oblique rectangular prism	$(p\,;\pm q\,;\,r)$
The right oblique-angled prism	$(\pm p\,;\pm q\,;\,r,$
The right rhombic prism	$(\pm p, \pm q\,;\,r)$
The right square prism	$\left(\begin{matrix}+\\=\end{matrix} p, \begin{matrix}\pm\\\mp\end{matrix} q\,;\,r\right)$
The cube	$\left(\begin{matrix}+\\=\end{matrix} p, \begin{matrix}\pm\\\mp\end{matrix} q, r\right)$
The regular tetrahedron and regular octahedron	(p, q, r) $(p, p-q, p-r)$ $(q-p, q, q-r)$ $(r-p, r-q, r)$
The direct symmetrical tetrahedron and square octahedron	$(p\,;\,q,\,r)$ $(p;\,p-r, p-q)$ $(q-r\,;\,q, q-p)$ $(r-q\,;\,r-p, r)$
The inverse symmetrical tetrahedron, and rhombic octahedron.	$(p\,;\,q\,;\,r)$ $(p\,;\,p-r\,;\,p-q)$ $(q-r\,;\,q\,;\,q-p)$ $(r-q\,;\,r-p\,;\,r)$

The regular triangular prism ; - $(p, q ; r)$
$(p, p - q ; - r)$
$(q, q - p ; - r)$

The rhombic dodecahedron ; - - $p, q, r)$
$(p, q, -\overline{p+q+r})$
$(p, -\overline{p+q+r}, r)$
$(-\overline{p+q+r}, q, r)$

A crystal may be represented by uniting the symbols of the planes of which it is composed. And it will be convenient to represent by a figure in brackets thus (6), the number of faces which arise from each symbol. Also frequently the crystal has *parallel* planes; in which case one of them may be considered as a repetition of the other ; and the plane thus doubled may be indicated by writing a 2 before it. Thus the form of borate of magnesia, called by HAUY *magnésie borateé defective*, may be thus represented.

Primary ; a cube.

Secondary ; $2(3)(1, 0, 0) + 2(6)(\pm 1, 1, 0) + (4)(\pm 1, 1, 1)$

Indicating — a cube 2 (3) (1, 0, 0), formed by repeating each of the primary planes (1, 0, 0);

Modified by 6 pairs of planes $(\pm 1, 1, 0)$; truncating the edges ;

And by 4 planes truncating angles, which are not repeated.

Hence the opposite angles are not symmetrically affected.

The situation of planes with respect to each other, may be determined by assuming a certain point as the pole of the crystal, and measuring the latitude and longitude of the centre of the plane with respect to this pole. If we suppose an ellipsoid of which the three axes are as the three edges a, b, c of the primitive form, we may suppose secondary planes to

be in their natural position when they are drawn so as to touch the ellipsoid; and we may consider as the centre of the face, the point of contact. The latitude and longitude (μ and λ,) of this point, are given by the formulæ which follow.

In the rhomboid, the axis of the rhomboid being the axis of the crystal

$$\cos.\ \lambda \text{ varies with } \frac{2p-q-r}{\sqrt{(p^2+q^2+r^2-pq-pr-qr)}}$$

$$\sin.\ \mu \text{ ——— } \frac{p+q+r}{\sqrt{(p^2+q^2+r^2)}}$$

In the prism, the axis being the axis of the prism

$$\tan.\ \lambda \text{ varies with } \frac{q}{p}$$

$$\sin.\ \mu \text{ ——— } \frac{}{\sqrt{(p^2+q^2+r^2)}}$$

And hence the situation of the planes is known. Also if any of the planes, instead of touching the ellipsoid, be nearer to or farther from the centre of the crystal, the *order* of the planes will not be altered.

Having thus determined what planes are adjacent, we find the angles which they make, by the formulæ given Art. 8.

In the rhomboid $(p\,;q\,;r)(p\,;q\,;r)$ being the planes, θ their angle, and α the dihedral angle of the primary form,

$$-\cos.\theta = \frac{pp'+qq'+rr'-(p'q+q'p+p'r+r'p+q'r+r'q)\cos.\alpha}{\sqrt{(p^2+q^2+r^2-2pq+pr+qr\cos.\alpha)(p'^2+q'^2+r'^2-2p'q'+p'r'+q'r'\cos.\alpha)}}$$

This is true also for the tetrahedron, and for the right rectangular prism, making $\cos.\alpha = 0$. In the other cases we have a formula involving the three dihedral angles of the primary form.

We can also find the angles contained between any two edges by first finding the equations to the edges, and then employing a formula given, p. 125.

The inverse problem, knowing two dihedral angles of the secondary figure to determine the symbols of the planes, is resolved by the same formulæ. In the case where the angles made with the primary planes are given, we have a direct solution. In the other cases we find the indices of the symbol of trial; and if the limits of the present paper allowed it, it might be shown how we might, after some trials, proceed directly to find the law.

P. S. The greater part of the formulæ in the preceding pages were calculated before my notice was directed to a paper by Mr. LEVY, in the Edinburgh Philosophical Journal for April 1822. Mr. LEVY there employs the principle which is the basis of the investigations now given, viz. the mode of expressing a secondary plane by means of its equation to three axes coinciding with the edges of the primitive form. From this principle he deduces, with great simplicity, the law of a secondary plane in a particular case; viz. when the intersections of that plane with two known planes, are parallel to *their* intersections with two others.* In order however to deduce the general formula, a new and different series of theorems is necessary, as appears in the course of this paper.

W. W.

* It may be observed, that the result in this case is easily obtained from the formula in Art. 14.

Fig. 16.

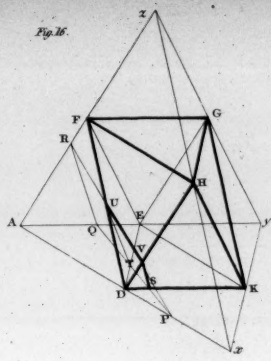


Fig. 17.

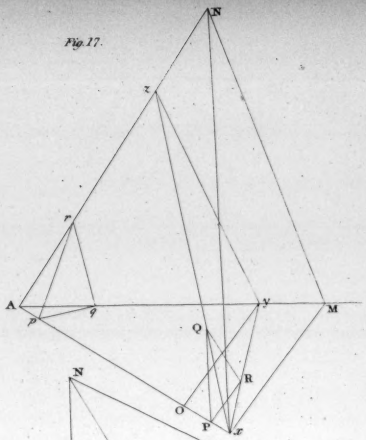


Fig. 18.

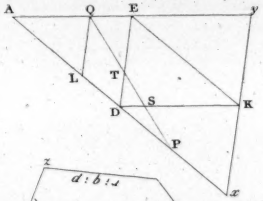


Fig. 19.

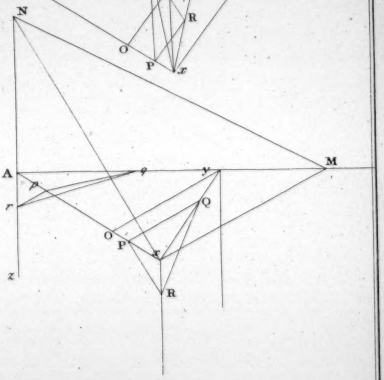


Fig. 22.

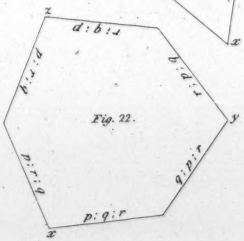


Fig. 21.

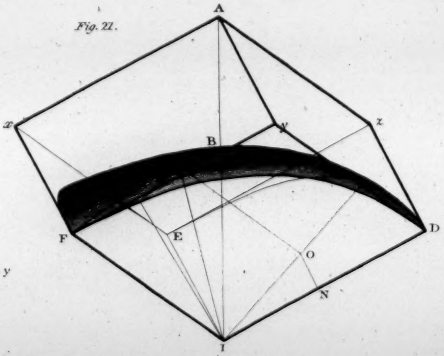


Fig. 20.

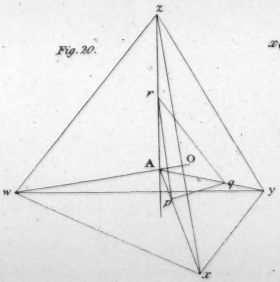


Fig. 6.

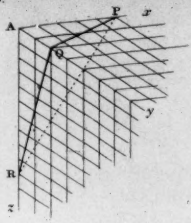


Fig. 7.

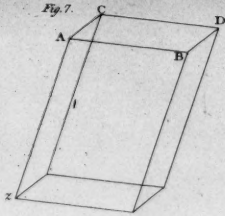


Fig. 8.

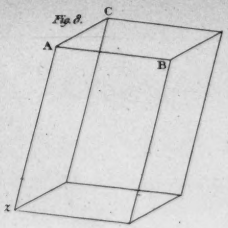


Fig. 9.

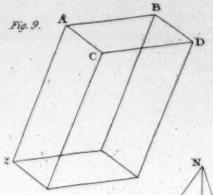


Fig. 10.

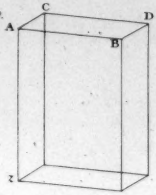


Fig. 11.

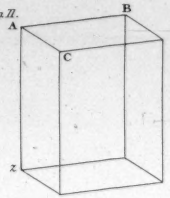


Fig. 12.

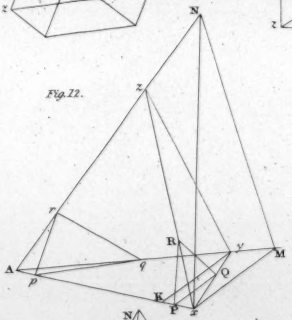


Fig. 15.

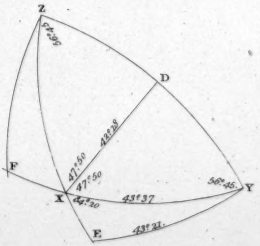


Fig. 14.

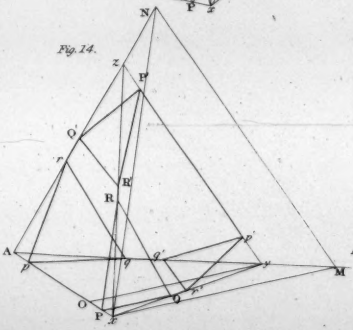


Fig. 13.

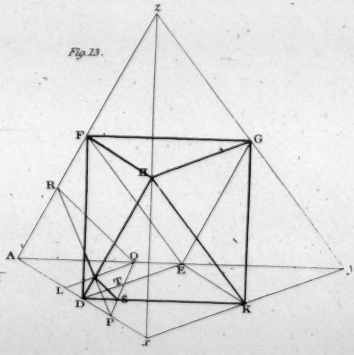


Fig. 1.

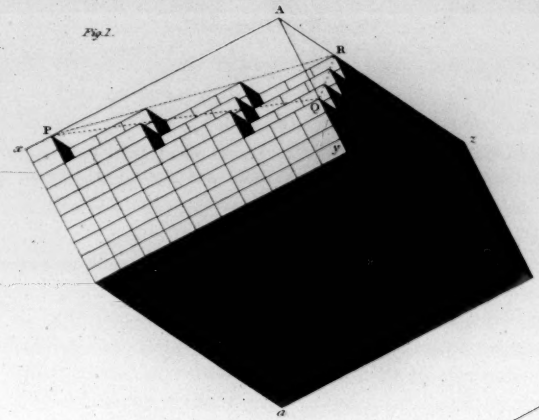


Fig. 2.

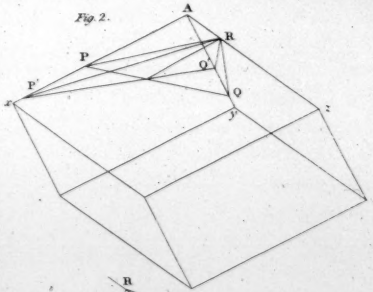


Fig. 3.

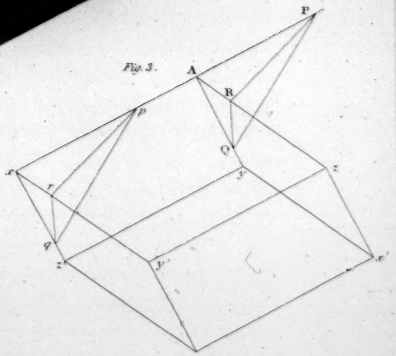


Fig. 4.

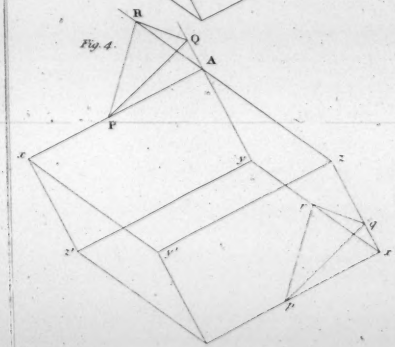
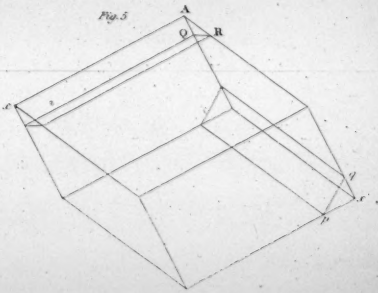


Fig. 5.



V. *Explanation of an optical deception in the appearance of the spokes of a wheel seen through vertical apertures.* By P. M. Roget, *M. D. F. R. S.*

Read December 9, 1824.

A curious optical deception takes place when a carriage wheel, rolling along the ground, is viewed through the intervals of a series of vertical bars, such as those of a palisade, or of a Venetian window-blind. Under these circumstances the spokes of the wheel, instead of appearing straight, as they would naturally do if no bars intervened, seem to have a considerable degree of curvature. The distinctness of this appearance is influenced by several circumstances presently to be noticed; but when every thing concurs to favour it, the illusion is irresistible, and, from the difficulty of detecting its real cause, is exceedingly striking.

The degree of curvature in each spoke varies according to the situation it occupies for the moment with respect to the perpendicular. The two spokes which arrive at the vertical position, above and below the axle, are seen of their natural shape, that is, without any curvature. Those on each side of the upper one appear slightly curved; those more remote, still more so; and the curvature of the spokes increases as we follow them downwards on each side till we arrive at the lowest spoke, which, like the first, again appears straight.

The most remarkable circumstance relating to this visual deception is, that the convexity of these curved images of the spokes is always turned downwards, on both sides of the wheel; and that this direction of their curvature is precisely the same, whether the wheel be moving to the right or to the left of the spectator. The appearance now described is represented in Plate XI. fig. 1.*

In order to discover a clue to the explanation of this phenomenon, it was necessary to observe the influence which certain variations of circumstances might have upon it; and the following are the principal results of the experiments I made for this purpose.

1. A certain degree of velocity in the wheel is necessary to produce the deception above described. If this velocity be gradually communicated, the appearance of curvature is first perceptible in the spokes which have a horizontal position: and as soon as this is observed, a small increase given to the velocity of the wheel, produces *suddenly* the appearance of curvature in all the lateral spokes. The degree of curvature remains precisely the same as at first, whatever greater velocity be given to the wheel, provided it be not so great as to prevent the eye from following the spokes distinctly as they revolve: for it is evident, that the rapidity of revolution may be such as to render the spokes invisible. It is also to be noticed that, however rapidly the wheel revolves, each

* The appearance in question has been noticed by an anonymous writer in the Quarterly Journal of Science (Vol. X. p. 282) who gives, however, no explanation of the phenomenon. It would have been impossible, indeed, to reconcile the facts as they are there stated, with any theory that could be imagined for their solution.

individual spoke appears, during the moment it is viewed, to be at rest.

2. The number of spokes in the wheel makes no difference in the degree of curvature they exhibit.

3. The appearance of curvature is more perfectly seen when the intervals between the bars through which the wheel is viewed, are narrow; provided they are sufficiently wide to allow of the distinct view of all the parts of the wheel in succession, as it passes along. For the same reason, the phenomenon is seen to the greatest advantage when the bars are of a dark colour, or shaded, and when a strong light is thrown upon the wheel. The deception is, in like manner, aided by every circumstance which tends to abstract the attention from the bars, and to fix it upon the wheel.

4. If the numbers of bars be increased in the same given space, no other difference will result than a greater multiplication of the curved images of the spokes; but if a certain relation be preserved between the angles subtended at the eye by the whole intervals of the bars, and of the extremities of the spokes, this multiplication of images may be corrected. The distance of the wheel from the bars is of no consequence, unless the latter are very near the eye, as in that case the apertures between them may allow too large a portion of the wheel to be seen at once.

5. If the bars, instead of being vertical, are inclined to the horizon, the same general appearances result; but with this difference, that the spokes occupying positions parallel to the bars, are those which have no apparent curvature: while the curvatures of the other spokes bear the same relations to these straight spokes, and to each other, that they did in the

former case. When the inclination of the bars is considerable, however, the images become more crowded, and the distinctness of the appearance is thereby diminished. The deception totally ceases when the wheel is viewed through bars that are parallel to the line of its motion.

6. It is essential to the production of this effect, that a combination should take place of a progressive with a rotatory motion. Thus, it will not take place if, when the bars are stationary, the wheel simply revolves on its axis, without at the same time advancing: nor when it simply moves horizontally, without revolving. On the other hand, if a progressive motion be given to the bars, while the wheel revolves round a fixed axis, the spokes immediately assume a curved appearance. The same effect will also result if the revolving wheel be viewed through fixed bars by a spectator, who is himself moving either to the right or left; because such a movement on the part of the spectator produces in his field of vision an alteration in the relative situation of the bars and wheel.

It is evident from the facts above stated, that the deception in the appearance of the spokes must arise from the circumstance of separate parts only of each spoke being seen at the same moment; the remaining parts being concealed from view by the bars. Yet since several parts of the same spoke are actually seen in a straight line through the successive apertures, it is not so easy to understand why they do not connect themselves in the imagination, as in other cases of broken lines, so as to convey the impression of a straight spoke. The idea at first suggests itself that the portions of

one spoke, thus seen separately, might possibly connect themselves with portions of the two adjoining spokes, and so on, forming by their union a curved image made up of parts from different successive spokes. But a little attention to the phenomena will show that such a solution cannot apply to them: for when the disc of the wheel, instead of being marked by a number of radiant lines, has only one radius marked upon it, it presents the appearance, when rolled behind the bars, of a number of radii, each having the curvature corresponding to its situation; their number being determined by that of the bars which intervene between the wheel and the eye. So that it is evident, that the several portions of one and the same line, seen through the intervals of the bars, form on the retina the images of so many different radii.

The true principle, then, on which this phenomenon depends, is the same as that to which is referable the illusion that occurs when a bright object is wheeled rapidly round in a circle, giving rise to the appearance of a line of light throughout the whole circumference: namely, that an impression made by a pencil of rays on the retina, if sufficiently vivid, will remain for a certain time after the cause has ceased. Many analogous facts have been observed with regard to the other senses, which, as they are well known, it is needless here to particularize.

In order to trace more distinctly the operation of this principle in the present case, it will be best to take the phenomenon in its simplest form, as resulting from the view of a single radius, (fig. 2.) OR of the wheel VW, revolving steadily upon its axis, but without any progressive motion, and seen

through a single narrow vertical aperture which is moving horizontally in a given direction PQ. Let us also assume that the progressive motion of the aperture is just equal to the rotatory motion of the circumference of the wheel. It is obvious that if, at the time of the transit of the aperture, the radius should happen to occupy either of the vertical positions VO or OW, the whole of it would be seen at once through the aperture, in its natural position; but if, while descending in the direction VR, it should happen to be in an oblique position RO, terminating at any point of the circumference at the moment the aperture has, in its progress horizontally, also arrived at the same point R, the extremity of the radius will now first come into view, while all the remaining part of it is hid. By continuing to trace the parts of the radius that are successively seen by the combined motions of the aperture and of the radius, we shall find that they occupy a curve R*a b c d* generated by the continued intersection of these two lines. Thus, when the aperture has moved to A, the radius will be in the position Oα; when the former is at B, the latter will be at Oβ, and so on.

Again; let us suppose that when the aperture is just passing the centre, the radius should be found in a certain position on the other side OY, and rising towards the summit. Then tracing, as before, the intersections of these lines in their progress, we shall obtain a curve precisely similar to the former. Its position will be reversed; but its convexity will still be downwards.

If the impressions made by these limited portions of the several spokes follow one another with sufficient rapidity, they will, as in the case of the luminous circle already alluded

to, leave in the eye the trace of a continuous curve line; and the spokes will appear to be curved, instead of straight.

The theory now advanced is in perfect accordance with all the phenomena already detailed, and is farther confirmed by extending the experiments to more complicated combinations.

It readily explains why the image, or spectrum, as it may be called, of the spoke, is at rest, although the spoke itself be revolving: a circumstance which might escape notice, if the attention were not particularly called to it.

Since the curved appearance of the lines results from the combination of a rotatory, with a progressive motion of the spokes, in relation to the apertures through which they are viewed, it is evident that the same phenomena must be produced if the bars be at rest, and both kinds of motion be united in the wheel itself. For, whether the bars move horizontally with respect to the wheel, or the wheel with respect to the bars, the relative motion between them, and its effects, in as far as concerns the appearance in question, must be the same. The attention of the spectator should in both cases be wholly directed to the wheel, so that the motions in question should be referred altogether to it. Thus, in fig. 4, the real positions, at successive intervals of time, of the spoke A *a*, when the wheel is rolling on the ground in the direction AZ, are expressed by the lines A*a*, B*b*, C*c* and D*d*. While the spoke is in these positions, the portions of it really seen through the fixed aperture VW, are the parts α, β, γ, δ, the impressions of which, being retained upon the retina, and referred to the wheel when in its last position,

form the series of points *m*, *n*, *p* and *q*, in the curved spectrum *m* D.

That the attention may the more easily follow the wheel in its progression, it is necessary that its circumference be distinctly seen, and its real situation correctly estimated. Hence, although it be true, that by a sufficient exertion of attention the phenomenon may be exhibited by means of a single aperture, it is much more readily perceived, when the number of apertures is such as to allow the wheel to be seen in its whole progress. For this reason the phenomenon is very distinct in the case of a palisade. Each aperture produces its own system of spectra; and hence, when the apertures occur at short intervals, the number of the spokes is considerably multiplied; but if the intervals be so adjusted as to correspond with the distances between the spokes at the circumference of the wheel, the images produced by each aperture will coalesce, and the effect will be much heightened.

A mathematical investigation of the curves resulting from the motion of the points of intersection of a line moving parallel to itself, with another line revolving round its axis, will show them to belong to the class of QUADRATRICES, of which the one which touches the circumference of the inner generating circle is that which is known by the name of the QUADRATRIX OF DINOSTRATES. Such a system of curves is represented in fig. 3, where MC, CN are the generating radii, A the outer, and B the inner generating circles, and PQ the common axis of the curves.

All these curves have the same general equation, namely,

$$y = (b - x) . \text{tang. } x.$$

where the co-ordinates are referred to the axis at right angles to the vertical generating radii, and passing through the centre of their revolution: the basis b being measured on the axis from the point of its intersection with the curve to the centre: and x being the arc of the inner generating circle, as well as the abscissa.*

A wheel simply rolling on its circumference exhibits, when seen through fixed bars, only those portions of the curves which are contained within the inner circle; but when its motion of revolution is more rapid than its horizontal progression, as when it is made to roll on an axle of less diameter on a raised rail-way, then the remaining portions of the curves will be seen, and others, on the lower part of the wheel, having a contrary flexure, will also make their appearance. These are seen at FF in fig. 3.

If the spokes, instead of being straight, be already curved, like those of the Persian water-wheel, their form, when viewed through bars, will undergo modifications, which may readily be traced by applying to them the same theory. Thus, by giving a certain curvature to the spokes, as in fig. 5, they will at one part of their revolution appear straight, namely, where the optical deception operates in a direction contrary to the curvature.

The velocity of the apparent motion of the visible por-

* This equality between the arc and the abscissa is a necessary consequence of the progressive motion of the wheel being equal to the rotatory motion of its circumference: the former motion producing the increments of the abscissa; and the latter those of the arc of the circle. The equation $y = (b - x) . \text{tang. } x.$ is deduced from a simple analogy of the sides of similar triangles.

tions of the spokes is proportionate to the velocity of the wheel itself; but it varies in different parts of the curve: and might therefore, if accurately estimated, furnish new modes of measuring the duration of the impressions of light on the retina.

Fig. 1.

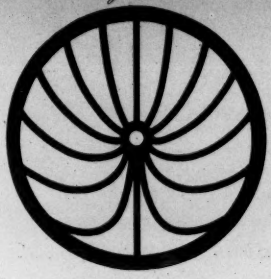


Fig. 2.

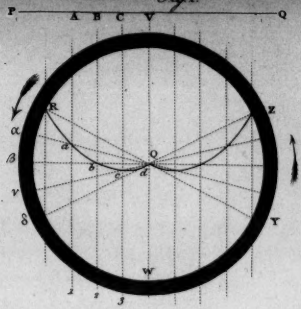


Fig. 3.

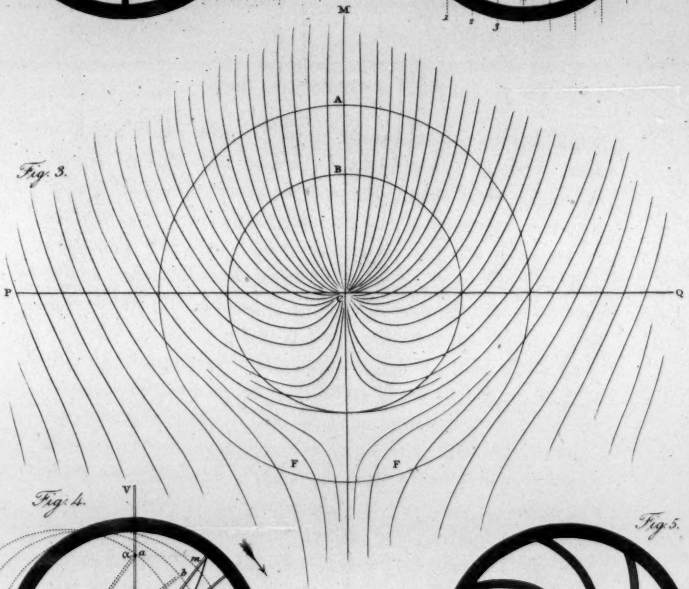


Fig. 4.

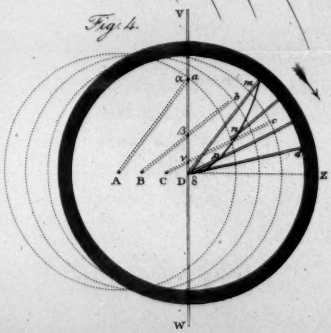
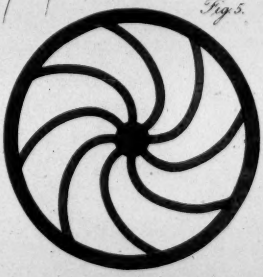


Fig. 5.



F. Bonte, sc.

VI. *On a new photometer, with its application to determine the relative intensities of artificial light, &c. By* WILLIAM RITCHIE, *A. M. Rector of the Academy at Tain. Communicated by the President.*

Read December 16, 1824.

1. IN a paper which I lately communicated to the Edinburgh Philosophical Journal, I endeavoured to show, that caloric flies off from the surface of a heated body by the repulsive energy existing between its own molecules, and consequently, that their velocity increases with the temperature of the body. This conclusion I deduced from the fact, discovered by DELAROCHE, that invisible caloric freely permeates very thin plates of glass, in the same manner as light, but that it is completely intercepted by thicker plates. If the temperature of the body be raised, the atoms of caloric will be brought nearer each other, their repulsive energy augmented, their velocity increased, and consequently, they will now find their way through a plate of glass which formerly intercepted them. If the temperature of the body be raised still higher, the molecules of caloric will acquire a velocity sufficient to permeate the various humours of the eye, and produce an impression on the retina, or in other words, they will become light. From this view of the subject, I was naturally led to the invention of an instrument which would be affected by visible caloric or light, whilst

it would not be sensibly acted upon by invisible coloric, or heat.

In short, I was led to the invention of a photometer, which appears to be the most accurate and delicate which has yet been described. But though such were the theoretical views which led to the invention of the instrument, its perfection does not depend upon any peculiar theory of light and heat. It is founded on the axiom, that equal volumes of air are equally expanded by equal quantities of light, converted into heat by absorption by black surfaces: and also on the well established principle that the quantity of light diminishes as the square of the distance of the luminous source from the object on which it is received.

2. The instrument consists of two cylinders of planished tin plate from 2 to 10 or 12 inches in diameter, and from a quarter of an inch to an inch deep. One end of each cylinder is inclosed by a circular plate of the same metal soldered completely air tight, the other ends being shut up by circular plates of the finest and thickest plate glass, made perfectly air tight. Half way between the plates of glass and the ends of the cylinders, there is a circular piece of black bibulous paper for the purpose of absorbing the light which permeates the glass, and instantly converting it into heat.

The two cylinders are connected by small pieces of thermometer-tubes which keep them steady with their faces parallel to each other, but turned in opposite directions, and also serve to make the insulation as complete as possible. The chambers are then connected by a small bent tube in

the form of the letter U, having small bulbs near its upper extremities, and containing a little sulphuric acid, tinged with carmine. The instrument is supported upon a pedestal, having a vertical opening through the stem to allow the glass-tube to pass along it, and thus secure it from accidents.

A small scale divided into any number of equal parts, is attached to each branch of the tube. In the annexed figure, Plate XII. ABCD and EFGH are the cylinders, AB and FG the plates of glass. CD, EFG the ends shut up by the circular tin plates, the blackened paper is represented by the lines between AB, CD and EH, FG. The other parts will be obvious from the mere inspection of the figure.

3. The accuracy of the instrument evidently depends upon the perfect equality of its two opposite ends. To ascertain, if it be accurately constructed, place it between two steady flames, and move it nearer the one or the other till the liquid in the tube remains stationary, at the division of the scale at which it formerly stood. Turn it half round without altering its distances from the flames, and if the liquid remains stationary at the same division, the instrument is correct. To show the extreme delicacy of the instrument, place it opposite a single candle, and it will be sensibly affected at the distance of 10, 20, or 30 feet, provided it be of sufficient diameter, whilst it will not be sensibly acted upon at the same distance by a mass of heated iron affording twenty times the quantity of heat. In order to cut off effectually the influence of mere radiant heat, I sometimes use screens composed of two plates of glass, placed parallel to each other, with a quantity of water interposed.

4. Place the instrument between any number of steady lights whose intensities are known, as for example, between four wax candles opposite one end, and one candle opposite the other, and move the photometer till the fluid remain stationary at the division where it formerly stood, and it will be found that the distances are directly as the square roots of the number of candles; or in other words, that the intensities of the lights will be inversely as the squares of the distances. If gas lights be employed, having burners capable of consuming known quantities of gas in equal times, and the photometer be placed between them, so that the effect upon the air in each chamber shall be the same; it will be found that the quantities of gas consumed by each, will be exactly proportional to the squares of the distances of their respective flames from the ends of the photometer.

5. This instrument seems well adapted for determining the relative quantities of light given out by the combustion of coal and oil gas. Place the instrument as before between the two burners, and ascertain the relative intensities of the two lights, by squaring their distances from the adjacent ends of the instrument; ascertain the quantities of gas consumed by each of the burners in the same time; multiply these quantities by the squares of the respective distances, and the product will be the relative quantities of light, afforded by the gases. Let d be the distance of the coal gas light and d' that of the oil gas light; and let q be the quantity of coal gas consumed in a given time, and q' the quantity of oil gas consumed in the same time, then the intensity of the coal gas will be to that of the oil gas $q\,d^2$ to $q'\,d'^2$.

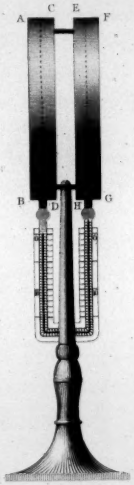
6. To find the ratio between the quantities of light given

out by the sun, and that afforded by a common candle, place one end of the instrument opposite the sun, and bring the candle opposite the other end, till the fluid in the stem remain stationary at the original division, and the light given out by the candle, will evidently be to that given out by the sun, as the square of a few inches to the square of the number of inches contained in 95,000,000 miles, provided none of the sun's light had been absorbed in its passage through the atmosphere. The delicacy of the instrument is such, that if it be placed opposite the sun without a counteracting force, the light absorbed from the body will be so great as to cause the liquid to move through a tube 20 or 30 feet long. By covering one end of the instrument, and directing the other to various quarters of the sky, we can ascertain the relation between the quantities of light reflected from the atmosphere, and clouds floating in those regions.

I am just now constructing a photometer about two feet in diameter, and two or three inches deep, with which I hope to appreciate the effect of heat in the feeble rays of the moon.

7. Though this instrument has some resemblance to Professor LESLIE's photometer, yet it is founded on principles essentially different. The one depending on the *difference* of the temperatures of the two bulbs, whilst the perfection of the other results from the equality of the temperature of the air contained in both chambers. The one has a scale a few inches long attached to one branch of the bent tube, whilst the scale of the other is the distance between the two antagonist flames. The delicacy of the one is, from its very

nature, confined within very narrow limits, whilst that of the other may be increased at pleasure. There are numerous problems which this photometer is capable of solving, and which, if this paper be favourably received, may form the subject of future communication.


A
C
E
F
B
D
H
G

VII. *The description of a floating Collimator.* By *Captain* HENRY KATER, *F. R. S.*

Read January 13, 1825.

THE line of collimation of a telescope is a line which passes through the centre of the object glass and the intersection of the cross wires placed in its focus. The apparatus which I am about to describe is intended to determine the situation of this line with respect to the horizon or the zenith, in some one position of an astronomical circle to which the telescope is attached.

A plumb line, a level, and an image reflected from the surface of a fluid, are the means which have hitherto been employed for this purpose. The defects and inconveniences of each of these have been felt, and the subject has for some years past engaged much of my attention, principally with a view of bringing instruments of portable dimensions into competition with those of a larger size.

Since a plumb line even of six feet in length would be subject to a deviation of only about three ten thousandths of an inch with an angular variation of one second, the application of the plumb line to small instruments becomes useless when great precision is required.

The difficulty of procuring a good and sensible level is well known, and though a very valuable instrument when

carefully fitted up, it is liable to so many errors from a variety of causes, that a single observation with the best level is little to be depended upon.

The method of observing by reflection, is perhaps by far the most perfect of any that has yet been practised; but it requires a union of favourable circumstances which rarely occurs. The fluid generally employed is mercury, which reflects a sufficient quantity of light to give a brilliant image, but is so easily deranged, that the slightest breath of air, not otherwise perceptible, or the distant passing of a carriage, the sound of which is scarcely heard, is sufficient to disturb its surface, and to render an observation either impracticable, or so erroneous as to be perfectly useless.

Nevertheless, by taking extreme care to protect the surface from wind, observations by reflection from mercury have been made with great success at the Royal Observatory at Greenwich, by means of the mural circle. But there are other objections besides those I have mentioned to all these methods. When an instrument with a plumb line is used, the observations may be conducted in two ways. In the first, the instrument being placed in the meridian, the plumb line is brought over a mark which is generally attached to the frame work carrying the microscopes. The star is taken as many nights in succession as may be thought necessary, and the instrument is then turned half round in azimuth, the plumb line brought over the same mark, and the star again taken, when the mean of the results of the readings in both positions of the instrument gives the altitude or zenith distance.

Here it is presumed, that the refraction remains the same

from night to night, and that the relative positions of the different parts of the instrument suffer no change, suppositions which are too gratuitous to be readily received.

In the second method, the plumb line being adjusted as before, the star is taken several times when very near the meridian, the time at each observation being noted. The instrument is then immediately turned half round in azimuth, readjusted if necessary, and the observations repeated, noting the time as before. The mean of the results of the readings in both positions of the instrument will be the altitude or zenith distance of the star at the mean of the observed times.

The time at which the star comes to the meridian being known, the difference between this and the mean of the times of observation gives the horary angle of the star with the meridian; whence the correction is computed which it is necessary to apply to the observed zenith distance in order to obtain the meridional zenith distance.

I know of no objection to this method except the length of time required for observing each star, the labour of computing the corrections, and the possibility of the instrument suffering a strain from being turned half round in azimuth.

The manner of using an instrument furnished with a level, is nearly the same as when it is constructed with a plumb lime; but as the level has a scale, the divisions indicated by the ends of the bubble are usually read off at the conclusion of each observation, and the resulting corrections applied to the observed zenith distance. This presupposes the value of the divisions of the scale to be well ascertained, and that they are equal, which last is not always the case.

In observing by reflection, the error which might arise

from turning the instrument in azimuth is avoided. The star is taken, and the telescope being afterwards directed to the reflected image, the very small movement in azimuth required to follow the star cannot well be supposed to occasion any strain; but the same observations of the time, and the same reductions to the meridian are necessary in this as in the preceding method.

If the instrument is fixed in the meridian, as is the case with the mural circle, the observation by reflection cannot be made on the same night as that by direct vision; and it may be supposed that unfavourable weather might occur for several nights and prevent the completion of the observation, during which interval a change in refraction, or in the relative positions of the circle and microscopes might take place, impossible to be detected, and which would vitiate the result. This inconvenience has been felt at Greenwich, and a second mural circle is nearly ready at the Royal Observatory, for the purpose of simultaneously observing the same star by reflection and by direct vision.

From the slight description that has been given of the different methods of observing, it must be evident, that the important desiderata are to keep the circle constantly fixed in the meridian, using no other motion than that of the telescope, and to possess the means of instantly determining and verifying at pleasure the place of the horizontal or zenith point with a degree of accuracy, limited only by the powers of vision assisted by the telescope.

If a telescope furnished with cross wires be adjusted to distinct vision upon a fixed star, the parallel rays proceeding from the object are converged to a focus, and an image of

the star is formed upon the cross wires. Conversely, if when the telescope is so adjusted, rays be supposed to pass from the cross wires through the object glass, they will emerge parallel, as if they had come from an object at an infinite distance. The cross wires are therefore similarly circumstanced with respect to vision, as a fixed star; and if another telescope, adjusted by a star, be employed to view these cross wires through the object glass of the former instrument, they will be seen with perfect distinctness, however near to each other the telescopes may be placed.

Professor GAUSS first published an account of this beautiful property, and he availed himself of it to measure the angular distance of the wires of a transit instrument by means of a theodolite placed near the object glass.

In the *Astronomische Nachricten*, *No.* 61, Professor BESSEL has given a "new method of determining the flexure of the telescope of astronomical instruments," which for elegance of invention can scarcely be surpassed. This he effected by means of the property described by Professor GAUSS; and at the end of his communication, M. BESSEL proposes a method of finding the zenith point of an instrument, of which the following is a translation.

"I may here be permitted further to remark, that the zenith point of an instrument may also be found without turning it in azimuth by a similar contrivance. For this purpose nothing more is required than a telescope, furnished with a sensible level, which may be placed on either side with respect to the axis. If this be placed alternately towards the north and the south, so that the bubble of the level may be similarly situated in both positions, then the mean of the

readings on the circle at each position of the telescope is the zenith point. This may be obtained in a manner most independant of all other corrections, if the level be fixed nearly at right angles to the axis of the telescope, suspending it in the zenith, and then repeating the observation after the telescope has been turned on its axis. This method, which presupposes only an arrangement very easily made in the slit of the observatory, or upon the pillars, will give a very exact result, since the cross wires appear at all times well defined and without motion, which is not always the case in the comparative observations of stars by a zenith sector and the circle itself."

By this method, the necessity of observing out of the meridian, or of waiting till another night for the completion of the observation is obviated, and the zenith point may be immediately determined with as much accuracy as can be attained by means of a level. But though this is by far the best mode of employing a level that has ever yet been devised, it is still subject to the objections which have been urged against that instrument.

It would require a level of very great delicacy and extent of scale to indicate the fraction of a second; and such a one would be readily deranged by a small inequality of temperature, or by the unavoidable elasticity of the parts necessary for its adjustment.

In the course of some former enquiries, I made many experiments to ascertain the degree of reliance that might be placed on the position of a body floating upon the surface of mercury, and fully satisfied myself that it might be so contrived as to have always when at rest the same

inclination to the horizon. I had thus a floating support to which I could attach a telescope, a support requiring no adjustment, offering the ready means of extreme accuracy, and precluding all fear of those errors which might arise from the use of a level.

For a preliminary experiment I procured a piece of oak, seven inches and a half long, four inches and a half wide, and one inch thick. Upon this, two wooden uprights in the form of Y s, were pinned and glued at the distance of five inches. Half way between these a small ring was screwed into the oak, and the telescope being laid upon the Y's, was firmly secured in its place by a string passing several times round it and through the ring.

In the middle of the longer sides of the oak support, and at right angles to its horizontal surface were inserted two pieces of brass, in which very smooth grooves were made, about one-tenth of an inch wide.

A deal box eight inches long, five inches wide, and an inch and a half deep, having its bottom just covered with mercury, received the float, which was kept in its situation in the middle of the box, and prevented from turning horizontally by two smooth iron pins passing through the sides of the box into the grooves. These were carefully regulated so as to allow the float to adapt itself with perfect freedom to the surface of the mercury.

The whole of the telescope was above the edges of the box, and a screen of black pasteboard, with an aperture equal to that of the object glass, was fixed to the end of the box. This is indispensably necessary, in order to exclude false light.

A fine achromatic telescope by Dollond, of thirty inches

focus, and two inches and three-quarters aperture, furnished with a wire micrometer, was placed upon a deal support with three legs. This support rested upon a flat stone laid upon sand which filled a pit of seven feet in depth, and afforded a perfectly steady foundation. The moveable wire of the micrometer was carefully placed in a horizontal position.

The floating collimator was put upon a table attached to the wall of the observatory, and was placed in the proper direction by looking through its telescope, and moving the box till the cross wires appeared upon the wire of the micrometer. The cross wires,* which formed an angle of about 15 degrees, were illuminated by a small lanthorn placed upon the table with a piece of oiled paper interposed.

My first trials were made with the telescope of a sextant, but as the object glass was not sufficiently perfect,† I did not conceive it worth while to register them, and I shall merely remark, that after deranging the float by moving it in a variety of ways, the cross wires returned, as nearly as the imperfection of the image would permit me to judge, to the same situation.

The telescope subsequently employed had an achromatic object glass of one inch and a quarter aperture, and seven inches and a half focus, and this gave a sufficiently good image.

In the experiments I am about to detail, every method I could think of was used that could fairly introduce error.

* The material which after numerous trials I found to answer best for cross wires, was the steel spring used in the balance of a watch.

† I may here observe, that I find this mode of examination to be a most severe test of the goodness of an object glass employed as a collimator.

The end of the box next the object glass of the telescope was raised so that the mercury ran from beneath the float to the other end of the box; it was then restored to its former position; this for the sake of brevity I shall designate by *O raised.*

The eye end of the box was elevated and restored to its place in like manner; this I have called *E raised.*

One side of the box elevated and replaced, *S raised.*

The other side of the box treated in like manner, *S s raised.*

The object end of the box raised, and the box carried in that position to the other side of the observatory, brought back and replaced, *O raised and carried.*

The eye end raised, and the box carried and replaced, *E raised and carried.*

S raised and carried, and *S s raised and carried,* signify similar operations, previously elevating one side or other of the box.

Out and replaced O first, signifies that the float was lifted quite out of the mercury and the box replaced, the end of the float next the object glass being brought first into contact with the mercury.

Out and replaced, E first, the same, with the eye end of the float first brought in contact with the mercury.

Out and replaced S first, the same with one side brought first into contact.

Out and replaced S s first, the same with the other side brought first into contact.

As soon as the cross wires appeared to be perfectly at rest, the angle was carefully bisected by the micrometer

wire, and the mean of seldom less than three readings registered.

The stability of the micrometer was severely proved by pressing upon the end of the telescope, and it was found that upon removing the finger, the micrometer wire always returned precisely to the same situation. The value of each division of the micrometer head is six-tenths of a second.

In the following tables, the first column contains the readings of the micrometer when the angle of the cross wires was bisected. The second column contains the difference between every two consecutive observations, and indicates the derangement of the position of the float from its having been moved between such observations. The third column contains the error in seconds, which would affect the determination of the horizontal point in consequence of the derangement of the float, and is equal to the half of that derangement.

Wooden Float.

	1st Set.	Divisions of Microm.	Difference.	Error in Sec^{s}. affecting the Horiz. point.
				″
Dec. 5	Previously to moving	83,9		
	O raised	85,4	+ 1,5	+ 0,45
	E raised	82,6	— 2,8	— 0,84
	S raised	80,9	— 1,7	— 0,51
	S s raised	79,3	— 1,6	— 0,48
	O raised and carried	79,5	+ 0,2	+ 0,06
	E raised and carried	70,9	— 8,6	— 2,58
	S raised and carried	71,9	+ 1,0	+ 0,30
	S s raised and carried	70,3	— 1,6	— 0,48

The image of the cross wires not being perfectly distinct, I limited the aperture to three-quarters of an inch, and thus

obtained a much better image. These observations were made in extremely damp weather, and the support had been kept for a few days in a very dry place.

	2d Set.	Divisions of Microm.	Difference.	Error in Sec^{s}. affecting the Horiz. point.
Dec. 8	O raised	31,6		
	After an hour's interval	16,6		
	Lanthorn close to wires	19,9	+ 3,3	+ $''$ 0,99
	After an hour	8,8		
	Out and replaced, S first	13,7	+ 4,9	+ 1,47
	Ditto, E first	13,1	— 0,6	— 0,18
	Ditto, O first	13,1	0,0	— 0,00
	O raised and carried	11,5	— 1,6	— 0,48
	S raised and carried	11,7	+ 0,2	+ 0,06
	Ditto, read again	10,7	— 1,0	— 0,30
	Carried box, kept as level as possible	5,2	— 5,5	— 1,65
	S raised and carried	3,5	— 1,7	— 0,51
	Out and replaced, S first	0,5	— 3,0	— 0,90

The wooden float being designed merely for preliminary experiments, and it not being my intention to introduce any errors but such as might arise from moving the float or agitating the mercury, I had a float made of cast iron eight inches long, four wide, 0,2 thick, and weighing 2 lb. 5 oz. troy. The telesccope was tied firmly to the Y s.

Experiments with the Iron Float.

	3d Set.	Divisions of Microm.	Difference.	Error in Sec^{s}. affecting the Horiz. point.
Dec. 9	Previously to moving	57,7		
	O raised	51,5		
	Not sufficient mercury; added more.			
	Out and replaced S first	47,1	— 3,8	— 1,14
	O raised and carried	43,3	+ 1,0	+ 0,30
	S raised and carried, escaped from the grooves	44,3	+ 2,3	+ 0,69
	Out and replaced, the surface at once in contact	46,6	— 0,1	+ 0,03
	Out and replaced, O first	46,5	— 2,3	— 0,69
	Out and replaced, E first	44,2	— 0,4	— 0,12
	Out and replaced, S first	43,8	— 0,7	— 0,21
	Out and wiped the mercury, S first	43,1	— 1,0	— 0,30
	Out and replaced, E first	42,1	— 0,2	— 0,06
	Out and replaced, S first	41,9	— 4,4	— 1,32
	Out and carried, S first	37,5	— 0,1	— 0,03
	Out and wiped, S first	37,4		
	On returning after an hour's interval	42,4	— 1,7	— 0,51
	Out and carried, S first	40,7	+ 2,0	+ 0,60
	Out and carried, S first	42,7	+ 0,4	+ 0,12
	Out and carried, S first	43,1	— 1,0	— 0,30
	Out and carried, S first	42,1	— 3,0	— 0,90
	O raised and carried	39,1		
	Agitated by raising and depressing the end of the box	No Var	— 1,1	— 0,33
	Out and carried, S first	38,0		
	Left the lanthorn close to the wires for three quarters of an hour and returned	39,3		
	Out and agitated mercury, S first	38,6	— 0,7	— 0,21
	O raised and carried	41,3	+ 2,7	+ 0,81
	Out and wiped mercury, S first	39,1	— 2,2	— 0,66
	Lanthorn taken away and returned in an hour and a half	45,0		
	Lanthorn left close to the wires, returned in an hour	50,8		
	Agitated by tapping the box	51,6		
	Returned in an hour	No Var.		
	Out and wiped mercury, S first	55,8	+ 4,2	+ 1,26

Experiments with the Iron Float.

	4th Set.	Divisions of Microm.	Difference	Error in Sec^{s}. affecting the Horiz. point.
Dec. 10	Out and replaced, S first	84,5		
	Ditto, and agitated the mercury, S first	90,3	+ 5,8	+ 1,74
	Ditto - - - ditto	89,8	— 0,5	— 0,15
	Ditto - - - ditto	88,7	— 1,1	— 0,33
	Ditto - - - ditto	87,3	— 1,4	— 0,42
	Ditto - - - ditto	84,9	— 2,4	— 0,72
	5th Set.			
	Out and agitated mercury, O first	84,3		
	Ditto - - ditto -	83,9	— 0,4	— 0,12
	Ditto - - ditto -	82,9	— 1,0	— 0,30
	Ditto - - ditto -	83,2	+ 0.3	+ 0,09
	Ditto - - ditto -	85,0	+ 1,8	+ 0,54
	Ditto - - ditto -	79,2	— 5,8	— 1,74
	An accident suspected			

The surface of the mercury being very dirty, it was carefully strained through a paper funnel.

	6th Set.			
	Out and replaced S first	81,0		
	Ditto - - ditto - -	80,9	— 0,1	— 0,03
	Ditto - - ditto . -	78,9	— 2,0	— 0,60
	Ditto - - ditto - -	78,5	— 0,4	— 0,12
	Ditto - - ditto - -	78,0	— 0,5	— 0,15
	Ditto - - ditto - -	77,0	— 1,0	— 0,30

The mercury carefully strained through a paper funnel, and the support oiled and rubbed dry.

Experiments with the Iron Float.

	7th Set.	Divisions of Microm.	Difference.	Error in Sec^{s}. affecting the Horiz. point.
Dec. 11	Out and replaced, S first	17,3		
	Ditto - - ditto	12,4	— 4,9	— 1,47
	Ditto - - ditto	15,8	+ 3,4	+ 1,02
	Ditto - - ditto	16,7	+ 0,9	+ 0,27
	Ditto - - ditto	10,7	— 6,0	— 1,80
	Ditto - - ditto	15,8	+ 5,1	+ 1,53

Experiments to show the effect of turning the box round.

	8th Set.			
	Previously to moving	16,7		
	Turned quite round	16,4	— 0,3	— 0,09
	Back again	19,0	+ 2,6	+ 0,78
	Turned round	15,8	— 3,2	— 0,96
	Back again	13,7	— 2,1	— 0,63

The micrometer wire not being *perfectly* horizontal, the cross was brought precisely to the same part at each experiment.

	9th Set.			
	Previously to moving	11,0		
	Turned round	10,3	— 0 7	— 0,21
	Back again	10,2	— 0,1	— 0,03
	Turned round	10,5	+ 0,3	+ 0,09
	Back again	8,4	— 2,1	— 0,63
	Turned round	8,1	— 0,3	— 0.09

The side and subsequently the whole float, whilst replacing, pressed gently upon the mercury.

	10th Set.	Divisions of Microm.	Difference.	Error in Sec^{s}. affecting the Horiz. point.
Dec. 11	Out and replaced, S first	95,7		
	Ditto - ditto	96,7	+ 1,0	+ 0,30
	Ditto - - ditto	99,5	+ 2,8	+ 0,84
	Ditto - - ditto	98,2	— 1,3	— 0,39
	Ditto - ditto	97,0	— 1,2	— 0,36
	Ditto - - ditto	97,7	+ 0,7	+ 0,21

The float pressed upon the mercury as before, and the mean of the readings of both angles of the cross wires taken.

	11th Set.			
	Out and replaced, S first	97,4		
	Ditto - - ditto	98,1	+ 0,7	+ 0,21
	Ditto - - ditto	100,5	+ 2,4	+ 0,72
	Ditto - - ditto	93,8	— 6,7	— 2,01
	Ditto - - ditto	95,0	+ 1,2	+ 0,36
	Ditto - - ditto	94,6	— 0,4	— 0,12

In the preceding experiments it may be seen that by far the greater number of the results are negative, or that the readings of the micrometer for the most part gradually decrease. I felt much at a loss to account for this, and at first supposed it to have been occasioned by the vicinity of the lamp to the Y supporting the cross wires; but I found on trial that this was not the fact; indeed in that case the effect would have been the reverse of what was observed. I can in no other way account for it than by supposing that as the weather was very damp and cold, my approach to the stand which supported the micrometer, caused the legs which were

next me to expand; a supposition which appears to be in some degree corroborated by the micrometer giving an increased reading on my return, after having been absent for some time from the Observatory. Whatever may be the cause, it constantly operates in one direction, and seems to be the principal source of the errors which are observable.

I now wished to ascertain whether by encreasing the length of the float, or by adding to its weight, the length being the same, I should attain greater accurary. I therefore procured two other cast iron floats, the one twelve inches long, four wide, and a quarter of an inch thick, and the other of the same dimensions as that before described, except that its thickness was half an inch, and its weight 4 lb. 8 oz. troy. Iron pins were fixed in the sides of these floats in place of the grooves, and grooves to receive the pins were attached to the sides of the box. The box in which both floats were used was fourteen inches long and six inches wide.

Before I made trial of the new floats, I browned that used in the preceding experiments by rusting it with nitric acid, and then rubbing it with oil; imagining that I might thus diminish any small affinity which the iron might have for the mercury. With the float thus browned, the following experiments were made.

Iron Float browned.

	12th Set.	Division of Microm.	Difference.	Error in Sec^{s}. affecting the Horiz. point.
Dec. 28	Out and replaced, S first	20,0		"
	Ditto - - ditto - -	20,5	+ 0,5	+ 0,15
	Ditto - - ditto - -	19,9	— 0,6	— 0,18
	Ditto - - ditto - -	19,0	— 0,9	— 0,27
	Ditto - - ditto - -	15,9	— 3,1	— 0,93
	Ditto - - ditto - -	20,3	+ 4,4	+ 1,32
	13th Set.			
	Out and replaced, O first	22,2		"
	Ditto - - ditto - -	15,8	— 6,4	— 1,92
	Ditto - - ditto - -	17,8	+ 2,0	+ 0,60
	Ditto - - ditto - -	23,5	+ 5,7	+ 1,71
	Ditto - - ditto - -	22,6	— 0,9	— 0,27
	Ditto - - ditto - -	16,4	— 6,2	— 1,86

I now made the following experiments with the long float, the rough surface of which had been made smooth by rubbing it with wax.

Long Float.

	14th Set.			
Dec. 29	Out and replaced, S first	31,0		"
	Ditto - - ditto - -	32,0	+ 1,0	+ 0,30
	Ditto - - ditto - -	34,8	+ 2,8	+ 0,84
	Ditto - - ditto - -	31,6	— 3,2	— 0,96
	Ditto - - ditto - -	37,3	+ 5,7	+ 1,71
	Ditto - - ditto - -	33,1	— 4,2	— 1,26

Experiments to show the effect of turning the box round.

	15th Set.	Divisions of Microm.	Difference.	Error in Sec^{s}. affecting the Horiz. point.
				$''$
Dec. 29	Previously to moving	33,3		
	Turned round, (much agitation)	32,5	— 0,8	— 0,24
	Back again	31,6	— 0,9	— 0,27
	Turned round	31,0	— 0,6	— 0,18
	Back again	31,2	+ 0,2	+ 0,06
	Turned round	30,4	— 0,8	— 0,24

I now laid aside the long float to try the short heavy float.

Short heavy Float.

	16th Set.			
				$''$
	Out and replaced, S first	57,2		
	Ditto - - ditto - -	59,7	+ 2,5	+ 0,75
	Ditto - - ditto - -	60,6	+ 0,9	+ 0,27
	Ditto - - ditto - -	56,1	— 4,5	— 1,35
	Ditto - - ditto - -	58,0	+ 1,9	+ 0,57
	Ditto - - ditto - -	57,6	— 0,4	— 0,12
	17th Set.			
				$''$
	Out and agitated the mercury, S first	67,0		
	Ditto - - dltto	65,5	— 1,5	— 0,45
	Ditto - - ditto	68,1	+ 2,6	+ 0,78
	Ditto - - ditto	64,7	— 3,4	— 1,02
	Ditto - - ditto	60,7	— 4,0	— 1,20
	Ditto - - ditto	66,0	+ 4,7	+ 1,41

The utmost care taken. The mercury carefully strained.

Short heavy Float.

	18th Set.	Divisions of Microm.	Difference	Error in Sec^{t}. affecting the Horiz. point.
Dec. 30 A. M.	Out and replaced, S first	32,1		"
	Ditto - ditto	34,9	+ 2,8	+ 0,84
	Ditto - ditto	34,7	— 0,2	— 0,06
	Ditto - ditto	33,2	— 1,5	— 0,45
	Ditto - ditto	34,2	+ 1,0	+ 0,30
	Ditto - ditto	31,8	— 2,4	— 0,72

Experiments to show the effect of turning the box round.

19th Set.			
Previous to moving	31,0		"
Turned round	33,2	+ 2,2	+ 0,66
Back again	33,9	+ 0,7	+ 0,21
Turned round	32,6	— 1,3	— 0,39
Back again	30,7	— 1,9	— 0,57
Turned round	31,3	+ 0,6	+ 0,18

Mercury carefully strained.

20th Set.			
Out and replaced, S first	21,1		"
Ditto - - ditto - -	27,0	+ 5,9	+ 1,77
Ditto - - ditto - -	22,1	— 4,9	— 1,47
Ditto - ditto -	25,7	+ 3,6	+ 1,08
Ditto - ditto - -	24,4	— 1,3	— 0,39
Ditto - ditto -	33,6	— 0,8	— 0,24

Great care taken that the side of the float should be in contact with the mercury its whole length, previously to putting down the surface.

	21st Set.	Divisions of Microm.	Difference.	Error in Sec'. affecting the Horiz. point.
Dec. 30 A. M.	Out and replaced, S first	25,1		″
	Ditto – ditto – –	24,1	— 1,0	— 0,30
	Ditto – ditto –	24,8	+ 0,7	+ 0,21
	Ditto – ditto – –	20,6	— 4,2	— 1,26
	Ditto – ditto –	23,1	+ 2,5	+ 0,75
	Ditto – ditto – –	21,6	— 1,5	— 0,45
	22d Set.			
Jan. 1	Out and replaced, S first –	25,9		″
	Ditto – ditto – –	19,4	— 5,5	— 1,65
	Ditto – ditto – –	21,9	+ 2,5	+ 0,75
	Ditto – ditto –	21,4	— 0,5	— 0,15
	Ditto – ditto – –	23,5	+ 2,1	+ 0,63
	Ditto – ditto –	21,9	— 1,6	— 0,48

Having caused the long float to be ground smooth, and browned it by rusting it with nitric acid, and rubbing it with oil, the oil was very thoroughly cleaned off, and the following experiments made with the greatest care.

Long float browned.

	23d Set.			
Jan. 2	Out and replaced, S first – –	1,4		″
	Ditto – ditto –	2,7	+ 1,3	+ 0,39
	Ditto – ditto – –	2,1	— 0,6	— 0,18
	Ditto – ditto –	3,4	+ 1,3	+ 0,39
	Ditto – ditto – –	3,3	— 0,1	— 0,03
	Ditto – ditto –	5,7	+ 2,4	+ 0,72

Long float browned.

	24th Set.	Divisions of Microm.	Difference.	Error in Sec^{s}. affecting the Horiz. point.
Jan. 3	Out and replaced, S first	51,1	— 1,7	— 0″,51
	Ditto – – ditto	49,4	— 0,7	— 0,21
	Ditto – – ditto	48,7	+ 1,5	+ 0,45
	Ditto – ditto	50,2		
	25th Set.			
Jan. 3	Out and replaced, S first	57,5	— 0,8	— 0″,24
	Ditto – ditto	56,7	+ 3,6	+ 1,08
	Ditto – ditto	60,3	— 2,9	— 0,87
	Ditto – ditto	57,4	+ 1,6	+ 0,48
	Ditto – ditto	59,0	— 0,9	— 0,27
	Ditto – ditto	58,1		
	26th Set.			
	Out and replaced, S first	63,3	+ 0,3	+ 0″,09
	Ditto – – ditto	63,6	— 2,5	— 0,75
	Ditto – – ditto	61,1	— 1,5	— 0,45
	Ditto – – ditto	59,6	+ 1,3	+ 0,39
	Ditto – – ditto	60,9	— 0,3	— 0,09
	Ditto – – ditto	61,2		

Of the one hundred and fifty one results in the column indicating the errors affecting the determination of the horizontal point, only twenty-eight are found exceeding one second; viz. one of 2″,58, another of 2″,01, ten between two seconds and one second and a half, and sixteen between a second and a half and one second; the remaining one hundred and twenty-three errors being all less than one second.

But as it is not to be supposed that an observer would be satisfied with a single determination, when he has the power in a very few minutes of attaining far greater accuracy, I shall

now show the effect that would result from using the mean of a few of the preceding observations.

	Wooden Float.	Error in Seconds.
1st Set.	Mean of the first four	— 0,34
	Mean of the second four	— 0,67
2d Set.	Mean of the first five	+ 0,36
	Mean of the second five	— 0,66
	Iron Float.	
3d Set.	Mean of the first five	— 0,16
	Mean of the second five	— 0,40
	Mean of the third five	— 0,02
	Mean of the last six	0,00
4th Set.	Mean of five	+ 0,02
5th Set.	Mean of five	— 0,31
6th Set.	Mean of five	— 0,24
7th Set.	Mean of five	— 0,09
8th Set.	Mean of four	— 0,22
9th Set.	Mean of five	— 0,17
10th Set.	Mean of five	+ 0,12
11th Set.	Mean of five	— 0,17
	Iron Float browned.	
12th Set.	Mean of five	+ 0,02
13th Set.	Mean of five	— 0,35
	Long Iron Float.	
14th Set.	Mean of five	+ 0,13
15th Set.	Mean of five	— 0,17
	Short heavy Float.	
16th Set.	Mean of five	+ 0,02
17th Set.	Mean of five	— 0,10
18th Set.	Mean of five	— 0,02
19th Set,	Mean of five	— 0,02
20th Set.	Mean of five	+ 0,15
21st Set.	Mean of five	— 0,21
22d Set.	Mean of five	— 0,18
	Long Float browned.	
23d Set.	Mean of five	+ 0,26
24th Set.	Mean of three	— 0,09
25th Set.	Mean of five	+ 0,04
26th Set.	Mean of five	— 0,16

On examining the above table, it appears that by taking the mean of a very few results, the greatest error, if the experiments with the wooden float be rejected, is four-tenths of a second, consequently the place of the horizontal point may be speedily determined by the use of the collimator, to the utmost degree of accuracy which the astronomical circle employed, is capable of attaining.

The results obtained by turning the collimator round without removing the float from the mercury, might have been expected to have been very nearly, if not wholly free from error; but as this does not appear to be the fact, and as the errors are all in defect, they seem to have been influenced by some constant cause, which, as before remarked, I believe to have been expansion of the stand of the micrometer in consequence of increased temperature.

When the float is removed in order to transport the box containing the mercury to the opposite side of the observatory, the manner of replacing it, so as to occasion the least error, seems to be that of bringing the edge of the side of the float first in contact with the mercury, and then gradually lowering it. This mode of removal can be necessary only when the collimator is used with a portable circle; but in a fixed observatory a plank should be laid, or a sort of railway contrived from one support to the other, on which the collimator should be either slid or passed along on rollers without removing the float from the mercury; by this arrangement the greatest, and perhaps the only source of error would be avoided.*

* It may perhaps be found preferable to have two boxes with mercury, and to carry the float from one to the other.

There appears to have been some advantage gained by using a longer float, and it certainly was improved by being browned, as previously to that operation, small particles of mercury were observed, occasionally to attach themselves to the float, which was not the case afterwards.

It may not perhaps be considered altogether superfluous to give in a few words the manner of using the collimator.

The instrument being placed on the north or south side of the observatory with its telescope pointed to the centre of the circle and nearly in its plane, it is to be directed, so that the wires of the telescope of the circle may be seen through it, when reciprocally the cross wires of the collimator will be visible through the telescope of the circle, and the collimator is to be so placed, that the cross wires may appear in the centre of the field of view. The place of the box should then be carefully marked, to ensure its being at once restored as nearly as possible to the same situation.

The collimator is then to be removed to the opposite side of the observatory, and the same process repeated, the situation of the box being here also carefully marked.

In observing, the star having been taken and the readings of the microscopes registered, the telescope is to be depressed to the collimator, and the angle formed by the cross wires carefully bisected. The collimator is then to be taken to the opposite side of the observatory, and the cross wires again bisected; the mean of the readings at the bisections will give the inclination of the collimator to the horizon, and the difference between this and the apparent inclination at either position of the collimator will be the correction to be applied to the mean of the readings registered at the bisection of the star.

For example, let the mean of the readings of the bisection of the cross wires when the collimator is to the south of the instrument be 7′.30″ of altitude, and when it is to the north 8′.40″. The mean of these readings 8′.5″, is the true inclination of the collimator to the horizon, and the difference between this and 7′.30″ (0′.35″) must be added to all altitudes taken to the south, or subtracted from those to the north of the zenith.

The instrument I have described may be called *the horizontal collimator*, but another and in most respects a preferable arrangement may be employed, similar to that suggested by Professor Bessel. The telescope may be firmly fixed in a position perpendicular to the float, and I should then name it *the vertical collimator*.* This must be placed directly under the telescope of the circle; and though not in a convenient position for observing, it yet possesses the very great advantage of obviating the necessity for carrying the collimator from one side of the observatory to the other, nothing more being requisite than to turn the float half round in azimuth, and to take the readings of the microscopes when the angle formed by the cross wires is bisected in each position of the collimator, the mean of which will be the place of the zenith point.

This is the construction which appears best calculated for a public observatory; but in addition, it would perhaps be adviseable that it should be furnished with a horizontal collimator, having a float of increased length. It is intended that

* The float of the *vertical collimator* should be circular, and an opening be made in the bottom of the tube of its telescope to throw light on its cross wires by means of an inclined plane mirror.

the horizontal collimator should remain stationary, and that the usual course of observations should be referred to it, its inclination to the horizon having been previously determined, and its permanency, when thought requisite, being examined by means of the vertical collimator.

As it is not necessary for the telescope of the collimator to have a tube, the object glass and the cross wires in the horizontal construction may be fixed in two uprights cast in one piece with the float. The distance of the object glass from the cross wires must be capable of the nicest adjustment. This may be effected by a screw cut on the outside of the tube in which the object glass is set, and a collar, by means of which after it is adjusted, it may be firmly secured in its proper place. There should be short pieces of tube screwed on each side of the upright, to protect the cross wires from injury, and also to contain the eye glass, which is convenient, as well for illuminating the wires, as for placing the collimator in the proper direction. This construction appears to promise the most perfect invariability of relative position between the line of collimation and the float. The box should be sufficiently deep to include the whole instrument, and should have apertures made in the ends, opposite to the object glass and to the cross wires. It is scarcely necessary to add, that it should also have a cover to exclude dust from the mercury, and a piece of ground glass or oiled paper should be placed between the cross wires and the lamp by which they are illuminated.

The accurate adjustment of the cross wires is a point of extreme importance. Upon whatever portion of an object glass parallel rays fall, they are converged precisely to the

same point in its focus, and consequently, whether the collimator be placed above or below the axis of the telescope of the circle, so long as the cross wires continue visible, the image will suffer no change of position. This affords an excellent method of discovering any want of parallelism in the rays; for if on placing the collimator as much above the axis of the telescope as possible, without losing sight of the cross wires, the image appears elevated above the horizontal wire, or if on placing it below the axis, the image appears to have descended, it is a proof that the rays falling upon the object glass of the circle are not parallel, but that they converge, and consequently that the cross wires of the collimator are too far from its object glass, and *vice versa*. It is necessary that this adjustment should be made with the utmost care.

It might possibly be supposed that the accuracy of the collimator would be augmented by increasing the length of its telescope, but this is not the case. It is the *direction* of a ray passing through the cross wires, and the centre of the object glass of the collimator, which is the subject of observation; and the direction of this ray is as definite in a telescope of an inch in length, as in one of ten feet focus. The degree of precision with which any variation in the horizontal inclination of this ray can be estimated, depends upon the length and power of the telescope employed to view the cross wires, and not upon the length of that of the collimator. There is an inconvenience however in using a telescope of too short a focus, as the cross wires are very much magnified, and consequently appear not so well defined; in addition to which, if a permanent point of reference be required, there might be some fear that the relative positions of the

float and telescope might suffer derangement, and the direction of the ray be consequently changed.

I may here point out an advantage, and not the least valuable, which this instrument presents, that of enabling the observer, by varying the inclination of the float, to bring a different part of the arc into use, and thus to check erroneous division of the circle: this may readily be done by securely fixing weights to either end of the float.*

I shall now proceed to give a description of the manner in which the floating collimator may be applied to the zenith tube.

The first zenith tube was I believe constructed for Dr. TIARKS. It was a telescope hanging in Y s upon pivots projecting from each side of the tube near the object end, and furnished with a wire micrometer: to this telescope a plumb line was attached.

When the star was upon the meridian, and of course sufficiently near the zenith to be seen in the telescope; it was bisected by the micrometer wire, and the divisions registered. The telescope was then inverted in the Y s, re-adjusted by means of the plumb line, and the following evening the star again taken, when the mean of the readings of the micrometer gave its zenith distance.

In the construction of the superb zenith tube 25 feet long, now making by Mr. TROUGHTON for the Royal Observatory at Greenwich, I understand it is intended that the axis of the tube shall be the centre of motion, and the plumb line be

* The above advantage may be considerably extended by the collimator being so constructed as to allow the inclination of the telescope to the float to be varied at pleasure.

suspended at the side. When the observations have been made a sufficient number of times with the plumb line on one side, the tube will be turned half round, and the observations repeated with the plumb line on the other side. The mean of both giving the zenith distance as before.

In this construction the zenith distance cannot be obtained in one evening; for were the telescope to be turned half round after the first observation, so much motion would be communicated to the plumb line, that there would not probably be time to re-adjust the instrument before the star would have passed out of the field of view.

As it is highly desirable that the completion of the observation should not be postponed, I endeavoured to effect this in a very fine zenith tube, which was constructed under my directions by Mr. DOLLOND for Colonel LAMBTON, and in another for Sir THOMAS BRISBANE, by placing the plumb line in the centre of motion; but these various forms are still subject to one or other of the inconveniences which have been detailed in the preceeding parts of this paper, and which it is the object of the floating collimator to remove.

The accuracy of the instrument I am about to describe, will depend upon the goodness of the telescope and of the wire micrometer employed. Exclusively of these it is within the reach of every observer, as the whole arrangement may be completed without difficulty and at a very trifling cost. Expence may contribute something in point of convenience, but can add nothing to its efficiency.

To a firm wall, at a sufficient height, let a shelf be fixed and supported by a bracket at each end. In the middle of this shelf let a circular aperture be made, rather larger than the object glass of the telescope. Precisely beneath, and at a

little distance from this aperture, the telescope is to be securely fastened to the wall in the direction of the zenith. This may perhaps be conveniently done by two irons driven into the wall, terminating in rings, into which the telescope may be passed and clamped. A box of sufficient size to contain the floating collimator being prepared with a circular aperture in the bottom of it, a very little less than that in the shelf, a piece of tube made of sheet iron, varnished brass, or even tinned plate, well painted or varnished, of a size to fit very tightly into the aperture of the box, must be passed into it and secured so as to project above the bottom on the inside an inch or two, and on the outside two or three inches more than the thickness of the shelf. The part of the tube outside the box must be passed through the hole in the shelf, and the box may then be readily turned about the axis of the tube as a centre. The side of the box being placed nearly in the direction of the meridian, its position must be determined by a pin driven perpendicularly into the shelf, so as to come in contact with a pin projecting from one corner of the box near the bottom, and in the direction of one of its sides. The box is then to be turned half round in azimuth, and a pin is to be fixed in the opposite end of the box in concontact with that in the shelf. By this contrivance, the box may be turned at pleasure half round the azimuth.

The float should be of cast iron, with a hole in the middle an inch larger in diameter than the tube. It is to be furnished with pins, and the box with corresponding grooves to steady it, as before described. An arm of plate iron is to be fixed to the float, its edge being at right angles to the surface. This arm is to project over the aperture, and to terminate in a small tube at the centre, to receive a telescope not

larger than that of a sextant furnished with crossed wires,* and having its object glass next that of the zenith tube.

To any convenient part, either of the shelf or of the wall, a support must be fixed, to which a circular screen of blackened tin may be attached by a joint, so as to be elevated to the vertical or lowered to the horizontal position at pleasure. In the centre of this screen a hole is to be made rather smaller than the telescope of the collimator. The screen is intended when in use to occupy a horizontal position, just above the crossed wires of the collimator, and to exclude false light from the object glass of the zenith tube. In order to illuminate the wires of the collimator, a small plane reflector, which may be of planished tin, is to be attached at a convenient angle to the upper side of the screen over its aperture. This may be made to turn stiffly upon a hinge to vary its inclination.

Having put a sufficient quantity of mercury into the box to enable the float to act freely, the screen must be turned up and the micrometer adjusted, so that a star may pass along its moveable wire. The screen being then restored to its horizontal position, the crossed wires of the collimator will be distinctly seen when the arm carrying its telescope must be bent till they appear in the centre of the field of view, and the telescope of the collimator must be turned in its tube till the opposite angles of the crossed wires are bisected by the micrometer wire. These adjustments may be considered as permanent.

To determine the zenith distance of a star, it must be

* A small black dot upon mother-of-pearl forms a very neat object instead of the crossed wires, but from some trials, I fear it cannot be made sufficiently small.

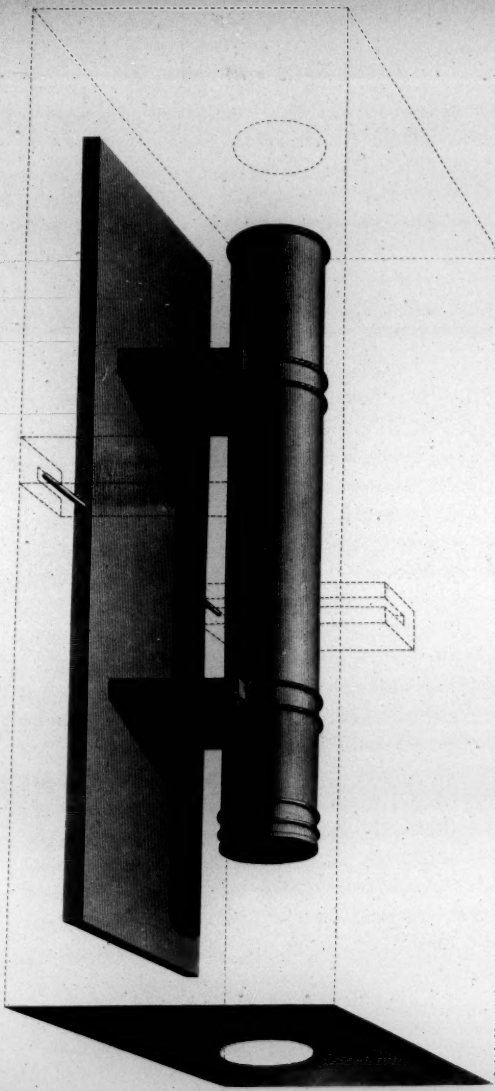
bisected by the micrometer wire at the time of its passing the meridian, and the division of the micrometer head read off and registered. The screen being then turned down, the angle formed by the crossed wires of the collimator is to be bisected, and the reading of the micrometer registered. The collimator is then to be turned half round, the angle again bisected by the micrometer wire, and the reading noted. The mean of these bisections is the place of the zenith, and the difference between this and the reading when the star was bisected, is the star's zenith distance.

It is evident that the operation for finding the place of the zenith may be repeated at pleasure, and consequently that the error, if any, in the zenith distance, may be ultimately referred to inaccurate bisection of the star, or imperfection of the screw of the micrometer.

I may remark, before I conclude, that a telescope,* similar to that I have used in the horizontal collimator, may be employed as a meridian mark for a transit instrument when a distant one cannot be obtained, and that the crossed wires afford an excellent object for the adjustment of the line of collimation. For this purpose, the telescope must be firmly fixed in the proper position. I attempted some years since to effect this by means of a convex lens, having cross wires in its focus, but as it did not occur to me to use an eye-glass, I was unable to place it in the proper direction, and after many unsuccessful trials I laid it aside.†

* The length of the telescope is here important to accuracy.

† Since this Paper was written, I have discovered that in the year 1785, such a meridian mark was actually used by Mr. RITTENHOUSE, who employed for the purpose the object glass of a telescope thirty-six feet long, in the focus of which was placed a metal plate, having several concentric circles drawn upon it. See Transactions of the American Philosophical Society, vol. ii.



VIII. *Notice on the Iguanodon, a newly discovered fossil reptile, from the sandstone of Tilgate forest, in Sussex. By* GIDEON MANTELL, *F. L. S. and M. G. S. Fellow of the College of Surgeons, &c. In a Letter to* DAVIES GILBERT, *Esq. M. P. V. P. R. S. &c. &c. &c. Communicated by* D. GILBERT, *Esq.*

Read February 10, 1825.

SIR,

I AVAIL myself of your obliging offer to lay before the Royal Society, a notice of the discovery of the teeth and bones of a fossil herbivorous reptile, in the sandstone of Tilgate forest; in the hope that, imperfect as are the materials at present collected, they will be found to possess sufficient interest to excite further and more successful investigation, that may supply the deficiencies which exist in our knowledge of the osteology of this extraordinary animal.

The sandstone of Tilgate forest is a portion of that extensive series of arenaceous strata, which constitutes the iron-sand formation, and in Sussex forms a chain of hills that stretches through the county in a W. N. W. direction, extending from Hastings to Horsham. In various parts of its course, but more particularly in the country around Tilgate and St. Leonard's forests, the sandstone contains the remains of saurian animals, turtles, birds, fishes, shells, and vegetables. Of the former, three if not four species belonging to as many genera are known to occur, viz. the crocodile, megalosaurus, plesiosaurus, and the iguanodon, the animal whose teeth

form the subject of this communication. The existence of a gigantic species of crocodile in the waters which deposited the sandstone, is satisfactorily proved by the occurrence of numerous conical striated teeth, and of bones possessing the osteological characters peculiar to the animals of that genus; of the megalosaurus, by the presence of teeth and bones resembling those discovered by Professor BUCKLAND in the Stonesfield slate; and of the plesiosaurus, by the vertebræ and teeth analogous to those of that animal.

The teeth of the crocodile, megalosaurus and plesiosaurus, differ so materially from each other, and from those of the other lacertæ, as be to identified without difficulty; but in the summer of 1822, others were discovered in the same strata, which although evidently referable to some herbivorous reptile, possessed characters so remarkable, that the most superficial observer would have been struck with their appearance, as indicating something novel and interesting. As these teeth were distinct from any that had previously come under my notice, I felt anxious to submit them to the examination of persons whose knowledge and means of observation were more extensive than my own; I therefore transmitted specimens to some of the most eminent naturalists in this country, and on the continent. But although my communications were acknowledged with that candour and liberality which constantly characterises the intercourse of scientific men, yet no light was thrown upon the subject, except by the illustrious Baron CUVIER, whose opinions will best appear by the following extract from the correspondence with which he honoured me.

" Ces dents me sont certainement inconnues; elles ne sont

point d'un animal carnassier, et cependant je crois qu'elles appartiennent, vu leur peu de complication, leur dentelure sur les bords, et le couche mince d'émail qui les revét, à l'ordre des reptiles. A l'apparence extérieure on pourrait aussi les prendre pour des dents de poissons analogues aux tetrodons, ou aux diodons; mais leur structure intérieure est fort différente de celles là. N'aurions-nous pas ici un animal nouveau, un reptile herbivore? et de même qu'actuellement chez les mammifères terrestres, c'est parmi les herbivores que l'on trouve les espèces à plus grande taille, de même aussi chez les reptiles d'autrefois, alors qu'ils etaient les seuls animaux terrestres, les plus grands d'entr'eux ne se seraient-ils point nourris de végétaux? Une partie des grands os que vous possidez appartiendrait à cet animal, unique, jusqu'à présent, dans son genre. Le tems *con*firmera ou *in*firmera cette idée, puisqu'il est impossible qu'on ne trouve pas un jour une partie du squelette réunie à des portions de machoires portant des dents. C'est ce dernier objet surtout qu'il s'agit de rechercher avec le plus de persévérance."

These remarks induced me to pursue my investigations with increased assiduity, but hitherto they have not been attended with the desired success, no connected portion of the skeleton having been discovered. Among the specimens lately collected, some however were so perfect, that I resolved to avail myself of the obliging offer of Mr. Clift, (to whose kindness and liberality I hold myself particularly indebted) to assist me in comparing the fossil teeth with those of the recent lacertæ in the Museum of the Royal College of Surgeons. The result of this examination proved highly

satisfactory, for in an Iguana which Mr. STUTCHBURY had prepared to present to the College, we discovered teeth possessing the form and structure of the fossil specimens.

In the annexed drawing, Plate XIV. examples of the recent and fossil teeth are represented, and the peculiar characters of each accurately shown; a description of it in this place will render the subsequent observations more intelligible.

Fig. 8 represents a portion of the upper jaw of the iguana viewed from within; it is magnified four diameters.

9 *a* shows the inner, and 9 *b* the outer surface of a tooth of the same, greatly magnified. It may be proper to remark, that the teeth differ considerably in the number of points, and that the eminence at *f*, fig. 9 *a*, is sometimes the first or second in the series, instead of being the third, as in the figure. In some teeth the points vary but little in size; they are more distinct on the edges of the teeth occupying the centre of the jaw, than in the anterior and posterior ones. The skeleton from which the drawings were made is three feet six inches in length. It is said to be the common edible iguana of the West Indies, but I have not been able to ascertain its species with certainty. The remaining figures represent different examples of the fossil teeth.

Fig. 1. *a* represents the outer, and fig. 1. *b* the inner surface of one of the largest and most perfect specimens of the teeth of the iguanodon. As the letters of reference in each figure indicate the same parts, they are explained here to avoid repetition.

a. Surface worn by mastication.

b. The serrated edges.

c. Fang broken; the cavity filled with sandstone.

d. Cavity or depression in the base of the fang, the effect of absorption caused by the pressure of a secondary tooth.*

e. Ridge extending down the front of the tooth.

Fig. 2. This tooth evidently belonged to a young animal; yet even in this example the apex is worn away, (see *a.* fig. 2 *c.*) The ridge extending down the front (see *e* fig. 2 *a.*) is more or less distinct in every specimen.

Fig. 3. A tooth much worn by mastication. The serrated edges and other characters are obliterated, the tooth being worn down to the point marked by the line at *g.* fig. 1. *a.* The fang has been removed by absorption; and the cavity formed by the pressure of the new tooth is very deep.

Fig. 4. In this specimen the point is perfect, and it therefore more closely resembles the recent tooth (fig. 9.) than those above described.

5. Is another example, where the point is but little worn.

6. A large strong tooth less curved than fig. 1 and 2. It probably occupied a place in the posterior part of the jaw.

7. In this figure, the cavity of the base of the fang for the reception of the new tooth is remarkably distinct.

The teeth above described, although varying from each other in some particulars, do not present greater dissimilarity than the differences arising from age, and the situation they respectively occupied in the jaw, would be liable to produce. Like the teeth of the recent iguana, the crown of the tooth is accuminated; the edges are strongly serrated or

* The hollow here described is so constantly found in every example, that it cannot be accidental. From the close resemblance it bears to the cavity formed in the base of the fangs of the recent iguana, by the secondary teeth, (Vide *d*, fig. 8) it may be confidently presumed that it is the effect of a similar cause.

dentated; the outer surface is ridged, and the inner smooth and convex; and as in that animal the secondary teeth appear to have been formed in a hollow in the base of the primary ones, which they expelled as they increased in size. From the appearance of the fangs in such fossil teeth as are in a good state of preservation, it seems probable that they adhered to the inner side of the maxillæ, as in the iguana, and were not placed in separate alveoli, as in the crocodile. The teeth appear to have been hollow in the young animals, and to have become solid in the adult. The curved teeth (figs. 1, 2.) probably occupied the front of the jaw; and those which are nearly straight, (fig. 3.) the posterior part.

It appears unnecessary to dwell longer on the resemblance existing between the recent and fossil teeth. Whether the animal to which the latter belonged, should be considered as referable to existing genera, differing in its specific characters only; or should be placed in the division of enalio-sauri of Mr. CONYBEARE, which includes marine genera only, cannot at present be determined. If however any inference may be drawn from the nature of the fossils with which its remains associated, we may conclude, that if amphibious, it was not of marine origin, but inhabited rivers or fresh-water lakes; in either case the term IGUANODON, derived from the form of the teeth, (and which I have adopted at the suggestion of the Rev. W. CONYBEARE) will not, it is presumed, be deemed objectionable.

It has already been mentioned, that of the bones of oviparous quadrupeds found in the sandstone of Tilgate forest, some are decidedly referable to the crocodile, and others to the megalosaurus and iguanodon; but our knowledge of the

osteology of the latter is at present so limited, that until some connected portion of the skeleton shall be discovered, it is impossible to distinguish the bones of the one from those of the other. Since, however, the teeth of the iguanodon are not known to occur in the Stonesfield slate, perhaps such of the bones from Tilgate forest as resemble those figured and described by Professor BUCKLAND, in Vol. I. Second Series of the Geological Transactions, may be attributed to the megalosaurus; while others not less gigantic may be assigned to the iguanodon. That the latter equalled, if not exceeded the former in magnitude, seems highly probable; for if the recent and fossil animal bore the same relative proportions, the tooth, fig. 1. must have belonged an individual upwards of sixty feet long; a conclusion in perfect accordance with that deduced by Professor BUCKLAND from a femur,* and other bones in my possession.

The vertebræ, as in the greater part of the fossil saurians, differ very materially from those of the recent iguana, crocodile, &c. They are not concave anteriorly, and convex posteriorly, but have both faces slightly depressed, resembling in this respect the vertical column of one of the fossil crocodiles of Havre and Honfleur. But among the recent lacertæ there are some, as the Proteus of Germany, the Syren of Carolina, and the Axolotl of Mexico, in which the vertebræ are deeply cupped at both extremities; and since the fossils in question are clearly of the saurian type, having the annular part united to the body of the vertebra by

* Vide Professor BUCKLAND's notice on the Megalosaurus. Geol. Trans. Vol. I. Second Series, p. 391.

suture, the discrepancy alluded to does not appear to be sufficiently important to invalidate the accuracy of the opinions which I have attempted to establish.

I have the honor to be,

SIR,

your most obedient Servant,

GIDEON MANTELL.

Castle Place, Lewes,
Jan. 1, 1825.

Teeth of the IGUANODON *a newly discovered FOSSIL ANIMAL from the Sandstone of TILGATE FOREST in SUSSEX.*

Portion of the Jaw of the Iguana, four times magnified.

Jn. Basire sculp.

IX. *An experimental enquiry into the nature of the radiant heating effects from terrestrial sources.* By BADEN POWELL, *M. A. F. R. S. of Oriel College, Oxford.*

Read February 17, 1825.

(1.) THE nature of the heating effect emanating from *luminous* hot bodies has been distinctly shown to be, in many particulars, very different from that evolved from *non-luminous* sources; but the ideas commonly entertained on the subject, are far from being precise and distinct. To gain if possible some ground for establishing more clear views, is the object of the following enquiries.

(2.) Professor LESLIE, in his well known and elegant experiments, (Inquiry concerning Heat, &c. chap. iii.) has fully established the theory of the effect of screens on radiant heat; and these effects give some of the most important criteria for examining the nature of radiating agents.

Those experiments apply only to the heat evolved from a non-luminous source. It therefore naturally becomes the subject in question, whether the interceptive power of glass is not limited to a certain temperature, or state, of the radiating source; and to this point accordingly the attention of several eminent observers has been directed in many well known investigations, among which those of M. DE LA ROCHE are justly regarded as the most important and complete. In these experiments it appears, that a greater effect

is produced on a blackened thermometer when a glass screen is interposed, in proportion as the body under trial approaches nearer its point of luminosity, or becomes more intensely luminous. (BIOT, Traité de Phys. tom. iv. p. 638.) (Ann. of Philos. O. S. vol. ii. p. 163.)

Both M. DE LA ROCHE and M. BIOT (See BIOT, iv. 612.) seem disposed to view the results obtained by the former upon the supposition of one simple agent, the principle both of light and heat. This is at first radiated as heat; at a certain point it begins to assume the form of light, when the interceptive power of glass decreases in proportion to the increase of luminosity.

(3.) As long as the hot body continues below the temperature of luminosity, the partial or total interception of the effect is precisely the same phenomenon as that described by Professor LESLIE in his experiments on screens and explicable in the same way; (Phil. Trans. 1816, Part I. On new Properties of Heat, Prop. 40.) And the apparent transmission of a portion of the effect must be referred to the same principle, as is clearly shown by Dr. BREWSTER, who has established, apparently beyond contradiction, the impermeability of glass to simple radiant heat upon quite independent principles.

(4.) Above the temperature of luminosity we must have recourse to further considerations. The hypothesis of M. M. DE LA ROCHE and BIOT appears to be nearly the same as that of Professor LESLIE (Inquiry, p. 162). And it certainly has the merit of simplicity and satisfactory explanation of the phenomena. But it is an opinion which has not received direct proof; and it is also obvious, that the phenomena may

be explained without it; for we may just as well account for the facts, by supposing two distinct heating influences, one associated in some very close way with the rays of light, carried as it were by them through a glass screen without heating it; the other being merely simple radiant heat, affected by the screen exactly as the radiant heat from a non-luminous body.

(5.) In order to ascertain which of these suppositions is true, it will not be sufficient to observe the effects produced by the intervention of a *screen alone*. We must combine this method with an examination of the relations of different sorts of heat to *surfaces*. These relations have been shown to differ according as the body is luminous, or not; in the one case, the direct heat affects bodies in proportion to the darkness of their *colour*, without regard to the texture of their surface: in the other, the magnitude of the effect depends solely on the *absorptive texture* without reference to colour. I use the term "absorptive texture," to signify that peculiar state of division in the particles of the surface, which has been shown, by Professor LESLIE and others to be most susceptible of the influence of simple radiant heat, and always to give a proportionally greater radiating power.

The question then is entirely one of facts; and involves no hypothesis as to the nature either of light or of heat. The object is simply to ascertain by experiment, whether, of the total heating effect radiated from a luminous hot body, the portion intercepted by a transparent screen is of the same nature as, or different from the part transmitted in its relations to the surfaces on which it acts.

(6.) In conformity with this view of the object proposed,

the general principle of the following experiments is this: taking different luminous hot bodies, to expose to their influence two thermometers presenting, one, a smooth black surface, the other an absorptive white one: thus obtaining the ratio of the total direct effect on the two, we may compare it with the ratio similarly observed, when a transparent screen is interposed.

(7.) This principle of experimenting was applied with one or two variations: and though in the abstract sufficiently simple, it will in practice require an attention to several considerations. I shall therefore proceed in the first instance to the detail of the different particulars; then give the results of the experiments in a tabular form; and lastly, recapitulate the conclusions, and make a few general remarks.

I. (8.) In the following set of experiments two common thermometers were employed. The diameters of their bulbs were, thermometer A, 0.6 inch,; B, 0.55. A, was coated with a wash of chalk and water, and B, with indian ink.

In order to compare the effects to be observed with those of simple radiant heat, I ascertained the ratio of the effects of the latter on the two bulbs thus coated, by a few preliminary trials; and found it to be very nearly one of equality, or perhaps the effect of the white rather greater than that of the black.

The two thermometers were graduated to quarters of centigrade degrees; and were both fixed on one mounting with their bulbs detached about one inch from its lowest part, and at the distance of about three-quarters of an inch from each other.

(9.) In the 2d set of experiments they were fixed into the

top of a box, the front of which was open, so that the glass screen could be applied to it or not, as required. When the screen was not used the box would acquire more heat, and radiate it to the bulbs in a small degree; which affecting them in the inverse ratio of their diameters, would diminish the ratio of their risings. That this diminution was very trifling, and not at all sufficient to account for the observed difference of ratio will be evident, because the 1st set was made without employing the box, the thermometers being suspended at a distance from any object which could radiate heat to them; and in this set the difference of ratio is quite as conspicuous. This remark applies likewise to the possible communication of heat by the air.

(10.) We must also take into consideration the effect due to the glass screen. When we consider the two bulbs as heated only by that part of the radiation which is transmitted through the screen, the screen may be regarded simply as a third body placed near the two bulbs; and whether it possesses a higher or a lower temperature, there will be a tendency to bring all three to an equality in proportion to the difference of temperature, and in the bulbs, dependent on their diameters modified by the state of their surfaces. This effect arises from simple radiant heat, whilst that derived from the luminous hot body, is evidently following a different law with regard to the surfaces. It will easily follow from what has been already shown, that such a secondary heating effect will be of a kind tending to *diminish* the ratio otherwise obtaining between the effects on the two bulbs. If the effect were of a cooling nature, the same thing would also take place: for I ascertained that the radiating powers of the

coatings employed, deduced from the observed rates of cooling, were in a ratio which happened to be almost exactly the inverse of that of the diameters; but this effect is probably always small, and I have roughly allowed for it, as will be seen immediately; taking the temperature of the screen by a small thermometer having its bulb in contact with the central part of the surface.

II. (11.) I now proceed to state the results, which will be most conveniently exhibited in a tabular form.

1st Set. Incandescent iron. Distance 7 inches.

Glass screen.

Experiment.	Rise of Thermometer in 1 min. centigrade.	
	A. White.	B. Black.
1	1.25	2.5
2	1.25	3.
Mean	1.25	2.75
Allowing for the screen (as below.	1.	2.5

No screen.

1	7.5	9.75
2	6.5	7.75
Mean	7.	8.75
Difference of exposed and screened results.	6.	6.25

(12.) Argand lamp without its chimney. Distance 3 inches.

Glass screen.

Experiment.	Rise of Thermometers in 1 min. centigradge.	
	A. White.	B. Black.
1	.75	1.75
2	.5	2.25
3	.75	2.25
Mean	.66	2.08
Allowing for the screen.	.41	1.83

No Screen.

1	1.75	3.5
2	1.75	3.25
3	2.	3.5
Mean	1.83	3.41
Difference of exposed and screened results.	1.42	1.58

(13.) II[d]. Set. Incandescent iron. Distance 6 inches.
Glass screen 2 inches from bulbs.

Experiment.	Temp. of screen before experiment by thermometer in contact.	Rise of Thermometers in 1 min, (centigrade.)		Temperature of screen after experiment.
		A. White.	B. Black.	
1	16.5	1.	1.5	25.5
2	16.5	.5	1.25	23.75
3	17	.5	1.5	24.5
4	17	.5	1.	22.25
5	17	.5	1.	22.25
	Mean	.6	1.25	
	Effect of the screen alone, heated above 25°.			
1		.25	.25	
2		.25	.25	
	The former result diminished for this effect.	.35	1.	

Incandescent iron. No screen.

1		3.	3.5	
2		3.	4.	
3		2.75	3.5	
4		3.3	3.75	
5		3.3	4.	
	Mean	2.95	3.75	
	Difference of the exposed and screened results.	2.6	2.75	

(14.) Flame of an Argand lamp without its chimney. Distance 3 inches.

Glass screen 1.5 inch from bulbs.

Experiment.	Temperature of screen before experiment.	Rise of Thermometers in 1 min. (centigrade.)		Temperature of screen after experiment.
		A. White.	B. Black.	
1	17	1.25	2.25	23.
2		1.25	2.25	
3		1.75	2.25	
4		1.25	2.75	
5		1.	2.25	
Mean		1.3	2.35	
Effect of the screen alone, heated above 25°.				
		.25	.25	
The former result diminished from this effect.		1.05	2.10	

Lamp. No screen.

1		2.25	3.	
2		2.5	3.25	
3		2.25	3.	
4		2.25	3.25	
5		2.5	3.5	
Mean		2.35	3.2	
Difference of the exposed and screened results.		1.3	1.1	

(15.) In these experiments it will be evident upon inspection, that the ratio of the effects produced on the white and black bulbs, is in every instance considerably greater when they were affected only by that part of the total heating influence which is transmitted through a transparent screen, than when they were exposed to the whole. This then would indicate, that on the removal of the screen some new heating power was brought into action which affected the ratio by the addition to each of its terms, of quantities in a ratio expressed by that of the difference of the exposed and screened results above given. This ratio is evidently one differing a little from equality, and agreeing nearly with that of the diameters of the bulbs inversely.

(16.) The experiments now detailed will probably be considered sufficient to substantiate the conclusion; but in researches of this kind, where great numerical precision is unattainable; it seemed desirable to give the experiments that confirmation which they wanted in point of intrinsic accuracy, by frequent repetition and variation. With this view I made a great number of trials with a large differential thermometer; the bulbs were about one inch in diameter and nearly three inches apart. The bore of the tube was about $\frac{1}{10}$ of an inch. Many of the experiments made with this instrument I shall not mention, as, although all agreeing to confirm my former conclusions, they were complicated by several unnecessary conditions.

(17.) In order to obtain results in the most simple manner, it was desirable to get rid of any action on one of the bulbs, and to expose only the other; the instrument thus acting simply as an air thermometer. The effects on each bulb, one

being painted with indian ink, and the other coated with white silk pasted on, when exposed, might thus be compared with those through a glass screen. I first tried the experiment by placing the bulb in the focus of a spherical tin reflector about six inches diameter; by this means the source of heat could be placed at a sufficient distance to preclude any effect from the glass screen.

(18.) The experiment was again varied by placing a large opake screen before the instrument, in which was an aperture through which one bulb might be exposed. To this aperture a piece of glass could be applied; each bulb was presented both with and without the glass.

(19.) In all these experiments it is evident, that any heating effect arising from the screen, would tend to diminish the ratio of the black and white effects; and this not being allowed for in the statement of the result, the difference between this ratio and that of the exposed effects will be in reality greater than appears.

The result are comprised in the following table.

(20.) Lamp. Bulb in the focus of a reflector.
Coatings white silk, and indian ink.

Experiments.	Screened.		Exposed.	
	White.	Black.	White.	Black.
1	5	8	11	15
2	4	9	13	15
3	6	11	12	14
Mean in 30 sec^s.	5	9.3	12	15

(21.) Incandescent iron.

1	4	7	11	13
2	4	6	10	10
Mean	4	6.5	10.5	11.5

(22.) Lamp. One bulb covered by an opake screen, the other exposed at an aperture. Dist. 5 inches. Screen 1.5 inch from bulb.

1	4	6	8	11
2	3.5	6	9.5	12
3	4	6	8	10.5
4	3	6	8	12
Mean in 1 min.	3.6	6	8.3	11.3

(23.) Incandescent iron.

in 30 sec^{s}.	3	4	12	9

(24.) It is perhaps not worth while to make any formal deductions from these results as to the ratios subsisting in the different cases. It will be sufficiently evident upon inspection, that when all due allowances are made, the ratio of the effects upon the white and black bulbs is considerably greater when they were affected only by the transmissible part of the heating effect, than when they were exposed to the whole. The part then which is added on the removal of the screen, is of a nature tending to add to the terms of the former ratio quantities in a ratio much nearer equality: quantities in a

ratio very nearly that which the effects of simple radiant heat would give.

III. (25.) I have above adverted to all the sources of error which occur to me as likely to have affected these results; and when these are taken into consideration, as well as the nature of the experiments and apparatus, the accordance which the different results exhibit, is perhaps as close as we can expect; and it appears that all the different sets of experiments agree in showing a very considerable difference in the *ratio* of the effects produced on a smooth black, and on an absorptive white surface, by that part of the radiant effect transmitted through glass, and by the total effect. If the total direct effect were the result of one simple agent, the intervention of the glass would, by intercepting some part of it, produce no other alteration than a diminution of intensity; the *ratio* of the two effects would remain unchanged. This distinction appears to me of some importance towards clearing our ideas respecting the nature of the phenomena, and thus affording an answer to the question originally proposed in reference to some theoretical views, which, though boasting the sanction of high authority, will be untenable if the validity of these results be admitted.

(26.) The general conclusions from all these experiments may be thus recapitulated:

1st. That part of the heating effect of a luminous hot body which is capable of being transmitted in the way of direct radiation through glass, affects bodies in proportion to their *darkness of colour*, without reference to the texture of their surfaces.

2d. That which is intercepted produces a greater effect in

proportion to the *absorptive nature of texture* of the surface, without respect to colour. These two characteristics are those which distinguish simple radiant heat at all intensities.

Thus then when a body is heated at lower temperatures, it gives off only radiant heat stopped intirely by the most transparent glass, and acting more on an absorptive white surface than on a smooth black one.

At higher temperatures the body still continues to give out radiant heat, possessing exactly the same characters.

But at a certain point it begins to give out light: precisely at this point it begins also to exercise another heating power distinct from the former; a power which is capable of passing directly through transparent screens, and which acts more on a smooth black surface than on an absorptive white one.

(27.) This last sort of heat, whatever its nature may be, is essentially different from simple radiant heat. It appears to agree very closely with what the French philosophers term "Calorique lumineux," and is, according to Professor LESLIE'S theory, a conversion of light into heat. These views of the subject are certainly gratuitous assumptions. We have no right whatever to identify those two agents, or to suppose that, because a heating effect very closely accompanies the course of the rays of light, the light is therefore converted into heat; but the theories above alluded to, seem to regard the *whole* heating effect of a luminous body as of this latter character. In this particular, the present inquiry has led us to an essential distinction; and if the experiments are to be relied upon, this peculiar sort of heat constitutes only *a part* of the total effect. These results do not indeed

present so simple a theory as that alluded to, but they apply very obviously to the explanation of many phenomena recorded by various experimenters.

(28.) The peculiar heat above spoken of, and which, for the sake of distinction and brevity, we may call "transmissible heat," is similar to that which acts in the solar rays, and which there constitutes the total effect. It is this kind of heat which has been employed as a principle of photometry, on the assumption that it is precisely proportional to the intensity of light. Within certain limits this may be the case; but there are unquestionably circumstances under which the relation is very different; such for example, as difference of colour in the light: and in general it cannot be assumed to hold good in light from different sources. To show this, there is a remarkable instance in incandescent metal, which produces but very faintly illuminating rays, yet its "transmissible heat" is very considerable. I have repeatedly tried the experiment with a small "photometer," having one bulb painted with indian ink and the other plain; the bulbs being in a vertical line; this instrument whether employed with or without its case, or a glass screen, always gave an effect of about 10° in 30″ at eight inches distance from a ball of iron heated to the brightest point in a common fire.

(29.) In making these last experiments, the effect was always greater when the instrument was used without its case, or a glass screen. This was no doubt in part owing to the greater action of the simple heat now admitted to the instrument on the coated, than on the plain bulb; but it was also in part occasioned by the circumstance, that the stem going to the upper bulb passes in contact with the lower,

and being a solid mass compared with the thin bulb, is slower in acquiring heat, and therefore cools it, thus increasing the apparent effect on the other.

(30.) In a variety of other experiments which I have tried, using either this "photometer," or another having the bulbs at equal heights, various apparent anomalies presented themselves, all which I found easily explained on the principles here established of two radiations, when connected with the various other considerations to which it is necessary to refer when employing instruments of this description; but I do not conceive it necessary to enter into any further details.

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